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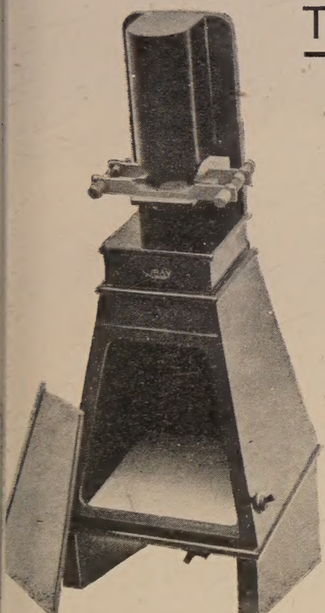

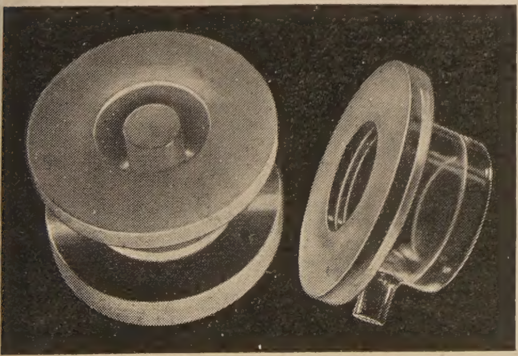

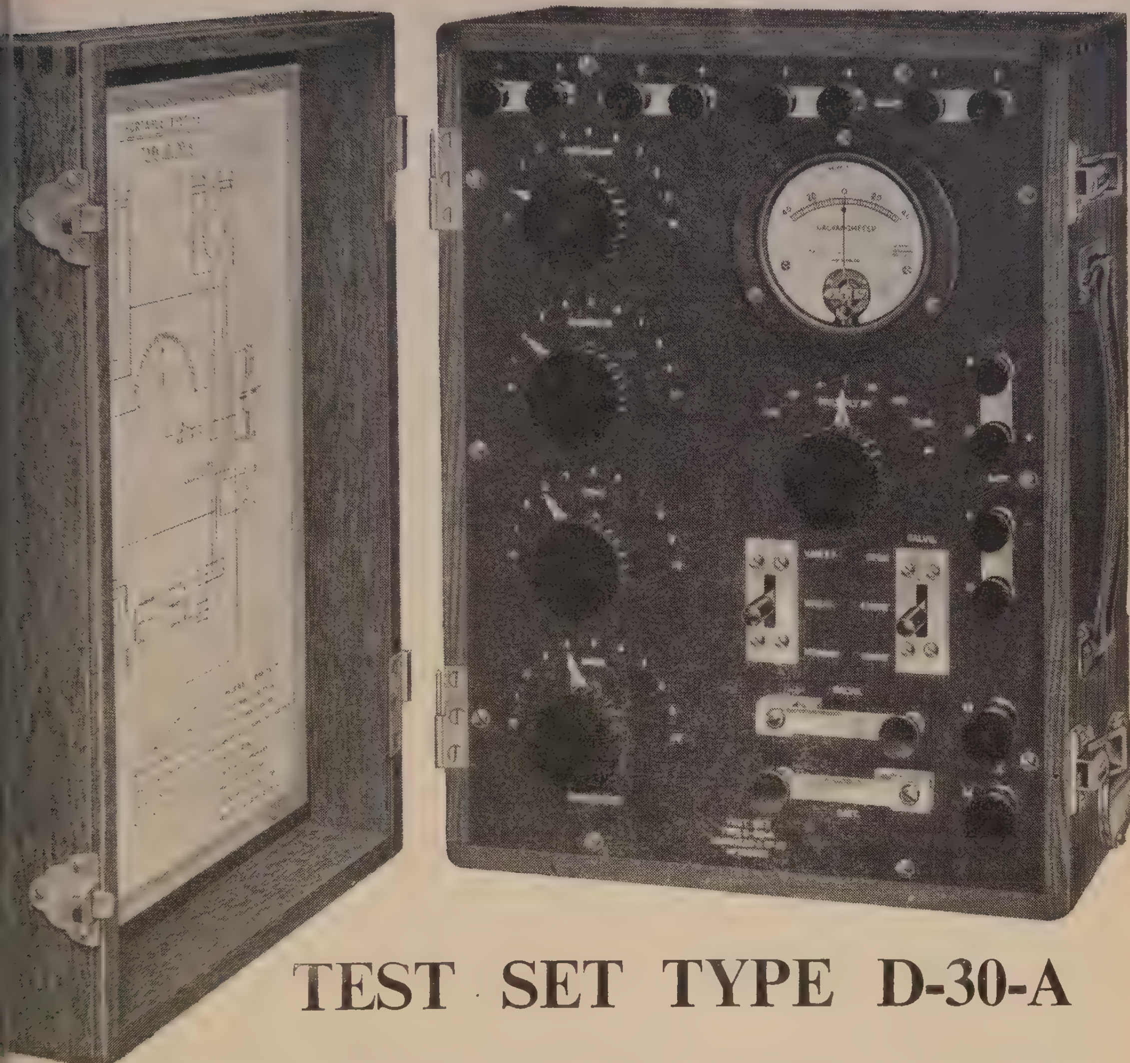

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PROBLEMS OF COLOUR MIXING IN THE DYEING INDUSTRY

BY J. G. GRUNDY,

Lecture delivered to the Colour Group, 25 *June* 1941

FOREWORD

THIS lecture has been compiled to indicate to the physicist the trend of thought of the practical dyer in his choice of dyestuffs. The lecture contains nothing new for the dyeing industry.

It must be understood that many of the terms used in the dyeing and textile industries have not necessarily the same meaning as when used by the physicist. To give an example, whenever the term " shade " is used in this paper, it is equivalent to the term " hue " as used by the physicist. Although there is a considerable looseness in the dyeing industry as regards terminology and expressions, it is not within the scope of this paper to clarify this position. This looseness in expressions can be indicated as follows:—

If two dyeings appear *similar* in all respects they may be described as accurate match, dead match, on shade or up to pattern. If a pattern apparently contains *less* dyestuff than another it is described as weaker, thinner, greedy, paler, empty, lacking in depth, lighter, starved, skimpy or " wanny ", whereas if a pattern apparently contains *more* dye than another it is described as too intense, heavier, fuller, stronger, darker, deeper, denser, overloaded, bursting or " brussen " with colour. All these terms are in common use in the dyeing trade.

[The lecturer gave a historical account of the art of dyeing from the earliest Chinese and Indian work, and the almost contemporaneous Egyptian art, specimens of which are found in the Valley of Tombs, to modern times. It is regretted that space does not permit the inclusion of this account, which mentioned, *inter alia*, that in the time of Pliny indigo was known only as a pigment and not as a dye, but that he knew of the use by the Egyptians of the polygenetic type of dyeing.—ED.]

The present-day dyer is rarely concerned with natural colouring matters and pigments. The introduction of synthetic dyestuffs in 1885, and the rapid progress by the dyestuff manufacturers in the production of dyes, revolutionized dyeing procedure.

In the years just previous to the present war the world production of synthetic dyestuffs amounted to seven hundred million pounds weight annually,

and most of this poundage was consumed by the textile industries. The Colour Index, which is the standard English dictionary on dyestuffs, describes over fourteen hundred distinct chemical compounds, nearly all of which find application in the textile and allied trades. It can be taken that most of these products are in everyday use. It will be shown later that the endeavours of the organic chemist to extend the ranges of dyestuffs are fully justified.

As reference will be made later to " primary colours ", it should be explained that, from a textile standpoint, as yellow cannot be produced by mixing any other dyes together, it has to be looked upon as a primary colour, and, therefore, in the dyeing and allied industries, red, yellow and blue are accepted as the three standard primaries. Owing to the wide differences in shade, fastness properties and methods of application of the dyestuffs available, however, the modern dyer has really to consider not three primaries, red, yellow and blue, but actually dozens of reds, yellows and blues.

Those not intimate with the industry will wonder why there should be the necessity for such a wide variety of red, orange, yellow, blue, brown, green and black dyes, when in theory one can produce every possible shade with the three primary colours, red, yellow and blue. There are several reasons for this. The selection of a dye is not dependent upon shade only, but on the type of material to be dyed and the fastness properties required. Dyes suitable for the dyeing of one fibre are not necessarily suitable for the dyeing of other fibres.

This point can be illustrated by Kiton Fast Green V on a cotton and wool union cloth. This colour dyes the wool and leaves the cotton undyed. A further example can be instanced: Chlorantine Fast Red K when applied on viscose rayon and acetate rayon, dyes the viscose rayon red and leaves the acetate rayon undyed. Again, dyes used for cotton are, with few exceptions, not applicable in commercial practice for the dyeing of wool, and only in very rare cases are dyes for wool used in cotton dyeing. Moreover, the dyeing procedure in the two industries is quite different. Only in a few branches of the industry are shades built up on the basis of red, yellow and blue. A large number of the shades dyed for the ladies' dress goods trade, and hosiery and carpet yarns, are obtained by using combinations of red, yellow and blue. The restriction in the use of red, yellow and blue is due partly to the availability of a large number of homogeneous dyes which yield orange, brown, green, violet and black shades. In general, these homogeneous dyes form a much more suitable basis for shades which are of the same character than combinations of the three extremes, red, yellow and blue. The shades obtained with the homogeneous oranges, browns, greens, violets and blacks can be adjusted with dyes of a similar tinctorial character, or with the primary colours, red, yellow and blue, as may be required. It should also be understood that as the reds, yellows and blues are themselves not pure in character, there are orange, violet and green dyes manufactured which cannot be matched for purity in shade with combinations of the purest primaries, red, yellow and blue.

This point can be clearly demonstrated by comparing a dyeing of Kiton Fast Green V on wool with the nearest shade of green to this obtained with a combination of a bright yellow and a bright blue dye. The combination of the two latter dyes yields shades very dull compared with the dyeing of Kiton Fast Green V.

It is essential for the dyestuff to possess fastness properties to meet the requirements of the dyed material from the point of view of subsequent finishing and making-up processes, and the expectations regarding the general utility of the finished garments.

[The lecturer then gave a detailed account of the fastness properties of various classes of dyes and continued :]

The dye manufacturers classify their dyestuffs into groups. For example, for cotton and regenerated celluloses of the viscose type, the groups are:—

1. Basic dyes.
2. Direct dyes, which are sub-divided as follows:
   (*a*) Direct dyes of average fastness properties.
   (*b*) Direct dyes fast to light.
   (*c*) Direct dyes which can be after-treated:
      (i) With formaldehyde to increase fastness to washing.
      (ii) With chrome and copper salts to increase fastness to washing and light.
      (iii) Those dyestuffs which can be diazotized on the fibre and developed with suitable naphthols or diamines, thereby producing shades fast to washing.
3. Sulphur dyes.
4. Azoic dyes.
5. Vat dyes.

For wool dyeing, the groups are:—

1. Acid dyeing dyes.
2. Neutral dyeing dyes.
3. The Neolan class of dyes.
4. Mordant dyes.

It will be appreciated that very few dyers have more than one trade for which to cater. For example, dyers in the cotton trade are dyers of loose cotton, cotton in hank, cheese or cop form, cotton piece goods or cotton knitted goods. In the case of the wool trade, dyers are either loose wool and slubbing dyers, or wool yarn or piece dyers. In addition, of course, there are many special branches of the dyeing industry, such as silk dyers and dyers of hose and rayons. Mention must be made of the fact that most dye houses have to consider the dyeing of unions in one form or another along with their normal run of trade.

Having probably only one class of dyestuff to consider, or, if more than one, closely allied classes, on account of the dyeing properties and the economics of

dyeing, the dyer must familiarize himself with the dyeing and fastness properties of the range of colours with which he is immediately concerned. He is, therefore, readily able to make a selection of dyestuffs for a specific job.

A consideration of the available dyes on the market for any particular branch of the dyeing industry is rather interesting. For example, in cotton dyeing there are at least sixty-two distinctly different red dyestuffs in use, as the firm with which we are associated actually market this number.

They can be classified as follows, according to the more important properties which are required in the various trades connected with cotton dyeing:—

Total number of dyes	62	Dyes fast to light and washing	17
Water-soluble dyes	43	Dyes fast to light, washing and chlorine	16
Dyes insoluble in water	19	Dyes fast to light and dischargeable	10
Dyes fast to light, five or over	20	Dyes fast to washing and dischargeable	15
Dyes fast to chlorine	17	Dyes fast to light, washing and dischargeable	7
Dyes fast to washing	26		
Dyes dischargeable with hydrosulphite	30		
Dyes which reserve acetate cellulose rayon	13		

[The lecturer then considered the differing requirements as regards fastness, according to the use to which cloth is to be put. Thus, men's hats, with their longer life and probable dry-cleaning, present different problems from women's hats. Suitings, linings, stockings, bathing costumes, curtains, carpets in houses, in hotels and on ships, and upholstery, all make different demands on the fastness to washing, to light, to perspiration, to sea-water, and so on. The lecturer then proceeded as below.—Ed.]

It will be appreciated that when selecting a dye for a specific purpose where certain fastness properties are required in the finished goods, the number of dyes suitable is considerably reduced. It is essential, of course, that the dye selected should give the correct shade, and this factor itself eliminates a large proportion of the available products.

There is no known method of determining by photometric or other scientific means how much colour will be required to obtain a match to a pattern and, if a combination of colours is required, what quantities of each colour. These factors have to be pre-determined by laboratory dyeing experiments or gained by experience in large-scale practice. In view of the fact that the laboratory dyeing is carried out in the same way as in large-scale practice, dyeing can be considered purely empirical.

These remarks are supported by the following:—

A. The solution of a dyestuff in process of application in many instances bears no resemblance to the shade of the finished dyeings. This is especially the case with dyes known as sulphur, azoic, vat and chrome colours.

B. The final shade with many dyes is only developed by chemical processes

operated in the actual dye-bath. The most important class of dyes in this category are the chrome fast colours, so called since they require chromium salts or chromates for the development of their shades. Without this treatment the shades produced by dyes of this group are, in general, of no commercial value, some being almost colourless.

C. The dyeing properties of individual dyestuffs irrespective of their shades or fastness properties vary considerably in their rate of dyeing and the degree of exhaustion under similar dyeing conditions. Therefore, in compound shades, when two or more dyestuffs are used, the number of variables is considerably increased.

D. Again, the physical appearance of the fibres to be dyed, and the structure of the material, play an important part in the resultant shades. For example, 2 per cent shades of the same dyestuff on two qualities of viscose rayon yield entirely different results. An extreme difference is obtained if the dyeings are carried out on filament and spun viscose, or on matt and bright acetate rayon.

E. Union fabrics containing two or more classes of fibres may be required to be dyed a full shade or a multi-coloured effect. It is quite common to have six to nine distinct dyestuffs in the one dye-bath.

The usual procedure of the dyer is to set out his day's work and to give instructions to the workmen in the dyehouse for the charging of the dye-baths. One dyer will control from 20 to 40 dyeing machines at the one time. His instructions are carried out by dyehouse labourers under the supervision of a foreman, depending upon the class of goods processed. As the dyeing proceeds, and reaches a certain stage, the workmen take samples from the bulk dyeings to the dyer for control. The dyer, after comparing the sample with the pattern he has to match, gives the workmen such instructions as may be necessary to adjust the shade. This procedure is continued until a satisfactory result is obtained. The dyer's match is nearly always subjected to a further examination by an independent member of the works staff, usually someone employed in the making-up room, before despatching to the customer. Even under this dual control it is not infrequent to have goods returned from the warehouse of the merchant or from the makers-up on account of inaccuracy in matching.

It is obviously essential, therefore, that people associated with the dyeing industry should be normal as far as colour vision is concerned. Normality of colour vision has for some time been tested and checked by the use of pseudo-isochromatic plates, first produced by Stilling. In more recent years Ishihara in Japan has also designed a series of plates to test for colour blindness. Within recent months the American Optical Company has prepared a further set of plates, enlarging upon the ideas of Stilling and Ishihara. These plates have proved to be very reliable as far as concerns the dyeing trade.

Different textiles are not all viewed in the same way when matching. Some patterns are viewed only by transmitted light, others by reflected light, whilst in some branches of industry both these factors have to be taken into con-

sideration. In the carpet and allied industries, where the cross-section of the fibre is the most obvious factor, dyeings under judgment are made into tufts, cut across to make a pompon, and compared on the brush-like surface. The intensity of the shade on the cut surface is much greater than on the yarn.

Since matching shades is relative rather than absolute, variations under different lighting conditions are experienced. Moreover, many patterns have to be matched on entirely different materials, using distinctly different chemical groups of dyestuffs. Having in mind such points, it is not surprising that disagreements arise as to what is a match, even amongst people who can be considered to have normal colour vision. This is particularly marked when dyeings are examined at different times, as in most cases daylight is taken as the standard source of illumination, and this is by no means a constant. Whenever possible, therefore, comparisons should be carried out on similar materials and under identical conditions.

Attempts have been made in recent years to arrive at some defined standard of artificial illumination with a view to eliminating the difficulties which arise in matching patterns under the varying conditions of daylight. In spite of a large amount of work which has been carried out in this direction, daylight is still generally accepted, a good north light being taken for preference.

It must not be overlooked that many shades of fabrics are used for evening wear, and chosen, not for their tone when examined in daylight, but for their tone under actual conditions of use, that is, in artificial light. For example, dance frocks, materials for stage wear, men's dinner and evening wear, must be considered mainly from this point of view. These factors, therefore, are specially watched by the dyer when dyeing and matching materials for these trades.

It has been stated that normal colour vision is only one-tenth as critical as known scientific instruments, and probably this statement is no exaggeration. A higher standard than normal vision for matching, however, would not be practical.

Matching to pattern in the textile trade is of high standard, but for reasons which have been described, a certain amount of tolerance must be allowed, otherwise few dye-houses could be run for profit.

From the foregoing notes the following conclusions can be drawn:—

1. Dyeing is purely empirical.

2. It does not appear possible to devise any type of scientific apparatus which could make due allowances for all the variables which occur in dyeing, and which could predict with even reasonable accuracy the percentage of dye or dyes to use for the production of a specific shade.

3. Instruments of the colorimetric type would be of little use, as large numbers of dyes in everyday use are quite insoluble in water. Others give almost colourless solutions.

4. Again, as demonstrated with a few chrome dyes on wool, the colour of

the solution bears little or no resemblance to the final shade obtained; and there are many similar classes of dyestuff which require some form of after-treatment to develop the correct shade.

5. In combination shades of two or more dyestuffs, it cannot be assumed that the dyes exhaust equally under the different dyeing conditions, and this factor alone would increase the difficulties of using some form of scientific instrument for measuring the amounts of colour required.

6. From an examination of the dyeings exhibited it will be seen that the shade obtained by any one dye is dependent to a considerable extent on the fibre itself. Similar percentages of one dye can yield weaker or stronger shades on different grades and qualities of cotton, wool, artificial silk, etc., according to the physical appearance of the fibre.

7. In a great number of cases it is not a single class of fibre which has to be dyed, but a mixture of fibres. This is particularly the case under present-day conditions, where there is a definite shortage of wool and cotton, and where artificial fibres are being used along with wool and cotton in ever increasing quantities. It is not uncommon in dyeings of this type to use six to eight different dyes on the union. This, again, rules out any possibility of devising a scientific method of predicting with accuracy the amounts of dye to use.

It will be readily seen from these conclusions that the modern dyer has to be a man of skill and resource if he is to cope with the large number of varying factors which can arise in connection with his work.

We hope this very brief outline of the dyeing industry has proved of interest, and that it will be agreed that theoretical consideration in regard to the quality and proportion of dyes for the production of dyed goods can only be of passing interest.

You may rest assured, however, that if by some means or other a way can be devised to make the work of the dyer easier, more efficient and effective, such a discovery will be more than welcome in an industry which contributes to such a large extent to the pleasures of the human race throughout the world.

In closing, I must express my indebtedness to The Clayton Aniline Company, Limited, for permission to read this paper, and the appreciable help rendered by Messrs. F. Parrott and B. Kramrisch in compiling it.

DISCUSSION

Dr. Cassie (Wool Industries Research Association). A great obstacle to appreciation of colorimetry by industry is the lack of any straightforward account of the subject. Much has been written and most of it is unintelligible even to physicists, unless they are specialists in the subject. The principles of colorimetry are now well established: it should be possible to state these clearly, and give logical deductions of the details necessary for practical work. When such an account is available, co-operation between physicists and colourists will become possible.

Mr. Warburton (Wool Industries Research Association). A remark in the discussion about people seeing colours differently as they age is important, as limiting the useful accuracy in matching samples dyed with different types of dye, since no two people have the same degree of retinal pigmentation. This is shown by the fact that although normal persons get very nearly the same co-ordinates for any colour when using a colorimeter—as a result of the convention that equal quantities of the primaries match white—it is very rare for the actual instrument settings to be the same for two people, even under identical conditions. Similarly, the proportions in which three primary dyes will be required to match a given sample (dyed with different dyes) will vary slightly from person to person, in much the same way as will the proportions required to obtain a match in different types of lighting, although the variation will be much less. A perfect match for all observers can thus only be obtained by dyes having similar absorption spectra.

The lack of sensitivity frequently complained of in connection with instruments such as the Donaldson colorimeter could, in many cases, be overcome by the use of a larger field without detriment to the accuracy of the instrument.

Mr. J. W. Perry. We owe our thanks to Mr. Grundy for the considerable care and thought evidenced by his paper, demonstrations and experiments, and it is clear that the problems of the dye-chemist and dyer are both varied and profound. From present indications I do not think it is too much to hope that dyeing may one day become an exact science, but it is clear that at present, in spite of much theoretical work on the subject, dyeing is an art. With the growth of knowledge, however, what is an art may, given certain conditions, become a science. In the dyehouse the control of the dyeing process is based largely upon the application of a form of trained instinct. and in this are applied, consciously or subconsciously, facts and experience which, if properly analysed and co-ordinated, would themselves form the substance of a not inconsiderable body of knowledge. Such a science, developed by research, would bring a comprehension of the real nature of the problems and would also facilitate the control of the process by making clear its essential nature.

The various interesting facts and phenomena described and illustrated by Mr. Grundy certainly indicate that there is considerable scope for such research which would so assist the dyer. It would be unnecessary and, indeed, impossible to draw a precise and fine distinction between chemistry and physics in their application to such a subject; in fact, the whole chemical aspect of the problem may be regarded also as a physical one. The older ideas of the dyeing process as depending upon solid solution or colloidal action may yet prove to be fruitful, but in view of the very wide variation in the nature of the phenomena involved, more ample scope for their explanation is to be sought in the recent views favouring chemical, physical or electrostatic adsorption. The physical liaison through the adsorption bond passes imperceptibly, with changing circumstances, into chemical combination, as is apparently also the case in dyeing.

As is indicated by recent work [Ruggli and Jensen, *Helv. chim. Acta*, **18**, 624 (1935); Zechmeister, Cholnoky *et al.*, *Chromatography*, London, 1941; and Brode and Brooks, *J. Amer. Chem. Soc.* **63**, 923 (1941)], valuable information may be obtained from the application of adsorptive analysis to dye compounds, using the methods of the Tswett chromatogram [Tswett, *Bot. Z.* **63**, 273 (1905)], and it may be that the behaviour of the dye in relation to the textile fibre in simple cases is itself subject to similar conditions and interpretations. Consideration of the phenomenon of dyeing from the point of view of the ζ-potential and by the aid of electrophoresis should also throw some light upon its essential nature, and assist the fundamental development of the subject. Some dyes would also no doubt require further special investigation owing to peculiarities in their behaviour, such as dichroism, polarization, etc., and consideration of these in classes related to their molecular structure is likely to yield information of great importance. Quite apart, however, from the more far-reaching value of such researches, it is probably not too much to say that specific differential spectrophotometric characteristics, by which the colour could be controlled and which could readily be found experimentally and tabulated in relation to certain conditions, would apply in all cases. In any given instance, therefore, given the necessary data (which could be obtained from previously recorded tests), the ultimate result could be reached with precision by the application of a definite spectro-photometric procedure, provided the essential conditions were known and controlled.

These remarks refer mainly to the chemical and physico-chemical aspects of the unsolved problems presented by Mr. Grundy, but there have also been important problems of a physical nature, and although it is not suggested that these are all completely solved, especially, as pointed out by Mr. White, in regard to the analysis of the complete reaction of the dyed fabric to incident light, yet it is important that those in the industries interested in colour should realize the great forward strides which have been made in this direction during the last quarter of a century. It is, indeed, unlikely that comparable progress has been made during the same period in any associated field. The physicist has here gone a good third of the way toward solving the dyer's problem, in that, out of a confused mass of unco-ordinated data and observations, the basic relationship of colour, as a visual function, to light, a form of energy, has been firmly established and formulated in generally acceptable terms. Thus the uncertainties of what is sometimes called the "psycho-physics" of colour have been removed from the arena, international agreement has been reached upon a system and an acceptable scale for designating colour in numerical terms and accurate colorimeters have been introduced, based upon the new system. This must, sooner or later, inevitably call for revision of outlook by all those interested in colour and colouring materials, for it provides the method and means for accurately specifying the main and ultimate purpose of the industries involved.

Colorimetric analysis has certainly required many years for its emergence

and standardization, however, and the dyer may perhaps be excused for failing to take it seriously until it affects him more nearly. Moreover, as indicated by Mr. Grundy, the dyer's colour-problem is really somewhat broader than is comprehended within the difficulties of colour-matching.

It was early recognized that colour is not directly related to physical constitution, but that it contains an important clue thereto; further, that if the chemist will think in terms, not of colour alone, but also of the absorption of light, important generalizations are made possible. This fact, which relates chemistry to spectrophotometry, is among those which, as Mr. Bunbury has remarked, have given the dye-chemist considerable power over the production of colour, representing a substantial and respectable achievement.

Equally the dyer may profit, now that the relationship of colour to physical stimulus has been firmly established. His problem, now less extensive and more clear-cut, may be regarded as consisting of (*a*) the production of substances having the requisite chemical, physical and tinctorial properties in relation to certain materials, and (*b*) fixing them, with or without modification of molecular structure, to textile fibres, again with due regard to physical and chemical conditions, so that the nett result is the production of an accurately specifiable physical reaction of the fibre to light. His problem may be to produce either a colour match in a specified illuminant or a spectrophotometric match, which ensures a colour match in all illuminants. This is an important distinction and really contains one more condition, which may be added as a refinement to the already seemingly formidable list of circumstances and conditions to be taken account of by the dyer and instanced by Mr. Grundy, viz., a specified degree of approach to spectrophotometric match; for upon this depends both the variation in colour which will take place when the illumination upon the dyed material is changed in character, for example, say from that of daylight to that of artificial light; and also the apparent differences in colour and matching properties, due to increased macular pigmentation in older observers. To achieve this condition in varying circumstances, however, precludes the possibility of using only three primary colours, whatever they may be, and it may be as well to emphasize that no definite number of colouring materials can be specified as being generally adequate to secure a reasonably exact spectrophotometric match, although in any given case it would require only a simple spectrophotometric test to ascertain what would enable the required conditions to be fulfilled.

May I add, in conclusion, that I think the wealth of problems still confronting the dyer, and very strikingly demonstrated by Mr. Grundy, afford ripe material for organized research, which I hope may some time be carried out, as well to benefit general knowledge, as also to relieve the perplexity of the dyer and to aid him in his work; but I hope also that the above remarks in appreciation of Mr. Grundy's paper will contribute to make it clear to those interested in applying physical methods to dyeing that the advances which have already been

made pave the way for a reconsideration of the dyer's art on modern lines and enable the processes which he uses to be more carefully analysed and controlled.

In regard to the question raised as to a suitable method of measurement to enable the accurate reproduction of a " black "; this is a photometric rather than a colorimetric problem, as the insensitivity of the eye to chromaticity differences at very low brightness-levels causes chromaticity differences to be of subsidiary importance. The first step should, I think, be to agree upon the colour which will be called " black ", since at best this will have some brightness and may also not be neutral; it will, in fact, be, physically, a pseudo-black. The light received from such a colour for different regions of the spectrum under controlled conditions of illumination may be measured against a standard, derived from the same source or from another operating in series and at the same efficiency as that source, by means of a low-brightness photometer used in conjunction with calibrated selective colour filters.

Dr. W. D. Wright. Mr. Grundy has given a very excellent survey of the problems that confront the dyer, a survey that is especially helpful to the physicist with little knowledge of the industry. He has made it abundantly clear that while colour is the first consideration, other factors, especially those of fastness, may determine which of a number of dyes is to be preferred for any particular purpose. It is, however, surprising to the physicist that since dyeing aims primarily at the production of colour, Mr. Grundy has not found it necessary to refer to the properties of a dye that give it its colouring power, namely, the extent to which the various parts of the spectrum are absorbed when light strikes a dyed surface. This action, it is true, is a physical action, but it is none the less fundamental to the business of dyeing. It is, in fact, interesting to speculate on how the dyeing industry would have developed if no dyeing had ever been carried out until the last few years. Some business magnate might have approached a group of scientists with the idea of establishing an industry for applying colour to materials. The scientists would have reported that colouring involved the absorption of light and was essentially a physical operation. A physicist might therefore have been appointed director of research, and he would no doubt have collected a number of physicists and chemists to assist him. Asked to produce a given colour, he would have advised the measurement of the absorption curves of certain dyes, and by a suitable combination would have endeavoured to produce the desired result. Before very long it would have become apparent that the practical problems of dyeing were mainly chemical, and the research director might have been asked to accept a more junior position. Nevertheless, the conception of the colouring aspect of dyeing would have been continuously linked up with the absorption curves of the dyes, and continual efforts would have been made to develop improved apparatus for measuring these curves.

This is pure fantasy, but it helps in deciding whether the physicist can now be of any service in the dyeing industry under the conditions in which it has

in practice developed, or whether he would merely be a nuisance. There seem to be two ways in which the physicist might be able to demonstrate his worth. In the first, if the dyer has any outstanding problems that he cannot solve, it may be that the physicist can help him to find the solution. In the second, it might be that if physicists undertook a long-term research into the colouring effects of dyes, they might be able to make fundamental recommendations that would revolutionize the art. This would certainly take time, and is purely speculative; but one would naturally expect the dyers to be extremely anxious to get to the fundamentals of their trade—and when they get there they will find physics. Unless they understand the physics of colour the result of their colour-mixing experiments must remain more or less of a mystery to them. It would be rash to claim that increased knowledge means increased dividends, but that is the frequent experience when scientific research is undertaken. I would, therefore, urge Mr. Grundy to consider whether research into the physics of colour as applied to dyeing ought not to be undertaken on a vastly greater scale than at present. I would not claim to have a detailed programme of research to offer, still less would I claim that all the apparatus needed for the research is available, but I do believe that an overwhelming case can be made out that the research should be attempted.

If a long-term programme is not approved, can the physicist prove his worth by helping with current problems? Mr. Grundy has referred to the case of a particular red dye that gives a more brilliant colour than any other. I would like to ask him whether he knows why this dye has this particular property? Does he believe that further chemical research is justified in trying to produce a still more brilliant colour of that hue, or has the maximum possible brilliance been obtained? Are there other colours whose brilliance might be increased, or is it the case that in general the most brilliant dyes possible are now available? These questions can be answered once the spectral absorption curves have been measured, but I do not know how else they can be settled. It is true that the curves may be of little direct use in helping to produce a new dye, but they will show whether time and money should be spent on the attempt, and they will show the properties the dye should, if possible, be given. I should be interested to know whether Mr. Grundy would agree with this point of view.

Another problem that appears to cause the dyer much trouble is that two samples may match in one light but not in another. This difficulty would disappear if the spectral reflection curves were of the same type—again, surely, a case for spectro-photometric measurements.

In the samples I showed at the meeting I raised the question of the colour that would result when the yellow and blue samples of slubbing were gilled together. As Mr. Grundy remarked, that is, in part at least, optical colour mixing, and from tests made before the meeting it certainly appeared that the physicist was able to prophesy the colour more accurately than the dyer. Yet

this is a process in constant use in the dyeing industry—another indication that use could be made of the principles known to the physicist.

I hope this discussion will provide some evidence of the value of increased co-operation between the physicist, chemist and dyer. If the Colour Group helps to develop this co-operation it will, I believe, be doing a very useful service.

AUTHOR'S REPLY. The contributions to the discussion have, in many instances, given emphasis to the complexity of the dyer's task, whereas from the physicist some hope is given that the difficulties in regard to the dyer's choice of dyes and quantities required for a given shade may eventually be more easily and more accurately controlled by the use of instruments not yet devised No doubt this possibility opens up a vast field of research, and in this regard the physicist must not lose sight of the fact that methods of scientific control must compete, not only in accuracy but in speed, with the empirical methods at present used by the dyer.

The physicist must also bear in mind that dyestuffs are technical products, and the dyer may from time to time have to use the same dyestuff manufactured by different dyestuff-producing firms, which, when dyed, yield similar shades to one another, but in regard to physical properties, that is, appearance of powder and appearance in an aqueous solution, vary very considerably.

It is not correct to assume that dyeing is wholly an art developed by years of practice and experience. The process of modern dyeing is more accurately described as an art well supported by scientific control. I see no reason to support the idea suggested by Dr. Wright that the dyer's colour-mixing experience is a mystery to him. The dyer is very intimate with his knowledge of colour-mixing as far as it is applied to the dyeing industry.

In regard to the possibility of two samples being a match in one light and not in another, Dr. Wright may be under some misapprehension, having regard to the remarks which he made. It was intended to refer to patterns which had been dyed with combinations of dyestuffs rather than to the behaviour of two or more individual dyestuffs.

It should be emphasized that there is no desire on the part of the dyeing industry to raise the standard of matching. This is already high enough, and most people associated with the dyeing and textile industries will readily agree that a higher standard than is generally maintained would not be a practical and economic proposition.

The discussion would indicate that there is a very wide gap in the trend of thought between the dyer and the physicist, and no doubt the narrowing of this gap by closer collaboration and understanding of extremes in point of view will be of considerable value towards the attainment of that goal which is the aim of the physicist.

THE ELECTROSTATIC COMPONENT OF THE FORCE OF SLIDING FRICTION

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ABSTRACT. For jerky motion due to relaxation oscillation to occur, the {friction, velocity} characteristic of sliding must exhibit a non-linear falling section and a minimum. A similar type of characteristic belongs to any electrical discharge device with a practically infinite resistance below a definite breakdown potential at which it becomes converted into a conductor. It is suggested that, due to contact electrification, the electrostatic component of the force of sliding friction assumes appreciable proportions when the boundary layer has dielectric properties ; thus cycles of slow charging and rapid discharging occur, and cause relative motion to proceed by cycles of slow sliding and rapid slipping when the average velocity, determined by the rate of propulsion of one friction element, is smaller than the velocity for which the frictional force assumes the smallest value. Experiments are presented to demonstrate that jerking decreases when the dielectric breakdown strength of the lubricating material is diminished, and that with the same lubricant jerking decreases as the rate of propulsion is increased. In these experiments, solid metal cylinders of 5·33 cm. nominal diameter were forced through hollow steel cylinders of slightly smaller bore, with various lubricants, in a 15,000-kgm. wt. recording Buckton testing machine which was operated slowly and at a uniform rate. The electric discharge current which accompanied jerking, even when *both* friction elements were made of steel, was observed when the friction elements were insulated from the Buckton machine and connected to a ballistic galvanometer. In accordance with the electrostatic interpretation of jerky motion due to relaxation oscillation, the surfaces remained undamaged and Amontons' law was obeyed when heavy jerking took place, whereas the relative motion of the two friction elements proceeded smoothly when the surfaces became severely torn, and the tangential force increased at a rate more than proportional to the radial force.

§ 1. INTRODUCTION

It is well known that slow sliding can proceed by a series of jerks exhibiting the two characteristics of relaxation oscillations*—a fixed amplitude, and a period determined by a relaxation time for the building up of a limiting

* A very readable account of relaxation oscillations has been published recently by G. F. Herrenden-Harker, 1940 (*Amer. J. Phys.* **8**, 1).

value. In the experiments described below, the amplitude of the jerks was observed to decrease when the dielectric breakdown strength of the material used as a lubricant was diminished, or when, with the same lubricant, the velocity of sliding was increased. Jerking was accompanied by an electric discharge through a galvanometer connected to the two friction elements. The expectation was confirmed that jerky motion was due to the characteristics of sliding and not to scoring. Amontons' law was satisfied, and the surfaces remained undamaged even when the tangential force of friction was a large fraction of the normal force, particularly when the jerks were very heavy. On the contrary, when severe tearing was observed, the relative motion between the two solid bodies had proceeded smoothly and did not obey Amontons' law. It is known from observations of contact electrification that an electrostatic force resisting sliding must exist if the boundary layer consists of dielectric or semi-conducting matter. The above observations support the view that this electrostatic component may be an appreciable part of the force of sliding friction.

§ 2. THE FORCE OF FRICTION AS A FUNCTION OF THE VELOCITY OF SLIDING

Under certain conditions, the sliding motion of elastically restrained bodies does not proceed smoothly, but by jerks; the friction elements appear to " stick together ", while the elastic force increases but is balanced by the correspondingly increasing force of *static friction.* When the elastic force reaches the limiting value of the force of static friction, static friction ceases to balance the elastic force, and a rapid slip suddenly occurs (figure 1). In these cases, for small

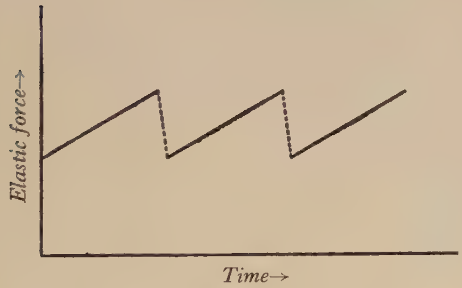

Figure 1

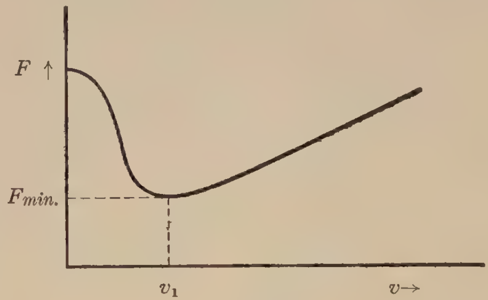

Figure 2

Figure 1. Force-time diagram when the average velocity of sliding is small and sliding friction is smaller than the limiting value of static friction. The two friction elements seem to stick together until the elastic force reaches the limiting value of static friction, when rapid slipping occurs.

Figure 2. Friction-velocity characteristic consistent with jerky sliding.

velocities, sliding friction is obviously smaller than the limiting value of static friction, and falls off with an increase in velocity. Haykin and his associates (1933, 1940) have pointed out that, for jerky motion due to relaxation oscillation

to occur, the {force of friction, velocity of sliding} characteristic of a particular frictional contact must exhibit the known characteristics of any mechanical or electrical device which is capable of performing relaxation oscillations under certain conditions, viz., a non-linear falling section and a minimum (figure 2). With this type of characteristic, two types of motion are possible, depending upon the uniform velocity v_0 at which one of the friction elements is moved forward. If this is given a value such that the representative point on the characteristic is to the left of the minimum ($v_0 < v_1$), the régime is unstable and oscillations occur. If the representative point lies on the rising arc to the right of the minimum ($v_0 > v_1$), the régime is stable and smooth sliding takes place.

Sliding proceeds by jerks when the régime is unstable. It begins when the elastic force reaches the limiting value of the force of static friction. On the falling section of the characteristic, the representative point upon reaching the unstable region flicks across almost instantaneously to the rising arc, which it follows towards smaller velocities, while the elastic force and the force of sliding friction both decrease. The representative point, upon reaching the unstable region near the minimum of the curve, again flicks almost instantaneously across to a small velocity (smaller than v_0), whence the elastic force increases until the limiting value of static friction is reached, and a new cycle begins. It is necessary to explain the given form of the dependence of frictional force upon sliding velocity, that is, the phenomenon of static friction and the falling section in sliding friction. This means that it is necessary to show which component of the force of friction has a characteristic with a non-linear falling section and a minimum, and to prove that under conditions of jerky motion this component represents an appreciable fraction of the frictional force.

§3. THE ELECTROSTATIC COMPONENT

Under certain conditions, electrical oscillations occur during sliding. It is well known that contact electrification arises whenever two different materials establish contact; when they are separated, they carry with them electrostatic charges of opposite sign (Schnurmann, 1941). When they are not separated, but made to slide one along the surface of the other, work must be done against the electrostatic attraction between the charges of opposite sign, if the surface layers have dielectric properties, so that the positions of the charges are fixed, and electrification follows processes of friction to the extent to which they involve making and breaking of contact. The separation of charges then increases as sliding proceeds until the potential reaches the breakdown value of the boundary layer, when this insulating layer suddenly becomes converted into a conductor, and a rapid discharge takes place. The {voltage, current} characteristic again has a non-linear falling section and a minimum, and (depending upon whether sliding is slow or rapid) the régime is either unstable or stable; that is, when sliding is slow it is accompanied by periods of slow charging and rapid discharge, whereas when sliding is rapid the discharge is continuous. Contact electrification

during sliding is established, and the occurrence of discharges is known, for instance, when a glass rod is being rubbed with a woollen cloth; the experiments described below were designed to demonstrate that under certain conditions this electrostatic component bears an appreciable ratio to the force of sliding friction. The implication of the electrostatic interpretation of jerky motion—that the cycles of apparent stick and slip should be cycles of slow sliding and rapid slipping (slow charging followed by rapid discharge)—could not be tested to the extent of measuring the small amount of slow sliding, because in this region the internal displacements and forces mask the displacements and forces between the surfaces in contact when one of the friction elements is elastically restrained, and static methods are applied.

§4. EXPERIMENTAL METHOD

Metal pins were forced through steel rings with various lubricants in a 15×10^3-kgm. Buckton single-lever testing machine with balancing spring and gear for recording the force and the movement of the cross-head. The cylindrical surfaces of the friction elements were finished by grinding.

Various methods of "cleaning" the surfaces before the application of a lubricant were tried. Good reproducibility was obtained when the surfaces were rubbed with several pads of cotton wool which had been soaked in petroleum ether before each experiment, and when care was taken that the surfaces did not become oxidized while they were not in use. A coat of wool-fat protected them satisfactorily. The "cleaned" surfaces were either flushed with a liquid lubricant or wiped with a grease. Care was always taken that a surplus of lubricant surrounded the leading edge. In the case of volatile liquids, a dropping arrangement maintained a ring of liquid around the leading edge.

From the record of the maximum value of the force of friction and the calculated radial force, the coefficient of friction was computed. When the pin has fully entered the bore of the ring, the radial force is

$$P_r = \pi \times d \times p_r \int_0^{l_h} dl,$$

where d is the diameter of the bore, and for friction elements of the same material

$$p_r = \frac{E \times S}{d} \frac{(D^2 - d^2)}{2D^2}.$$

(E = Young's modulus, S = interference, D = outside diameter of the ring). The length l_h of the ground surface of the ring was always shorter than l_s, the length of the ground surface of the pin, so that the radial force increased linearly from zero at the beginning of assembling to a value which remained constant over a length $(l_s - l_h)$ and then decreased linearly to zero during dismantling. With steel elements the radial force in the fully assembled condition was, for instance, $P_r = 31 \cdot 4 \times 10^3$ kgm. wt. for $S = 2 \cdot 54 \times 10^{-3}$ cm., since the diameters were throughout $d = 5 \cdot 33$ cm. and $D = 10 \cdot 67$ cm. In the case of a brass pin and

steel ring $P_r'=25{\cdot}9\times10^3$ kgm. wt. would be calculated in the fully assembled condition from

$$p_r'=\frac{e}{\frac{1}{E_1}\left[\left(\frac{D^2+d^2}{D^2-d^2}\right)+\sigma_1\right]+\frac{1-\sigma}{E}},$$

where $e=S/d$, E_1' and E are Young's moduli of steel and brass respectively, and σ_1 and σ are Poisson's ratios for steel and brass. The pins were tapered at one end to ensure good entry conditions, and the rings were given a small radius at both ends.

The envelope of the recorded force of friction during a complete operation of assembling and dismantling of the friction elements at a uniform velocity of the Buckton machine (constant average rate of propulsion of one friction element) was represented by a trapezoid, if Amontons' law was obeyed. Deviations from Amontons' law were recorded as a more-than-proportionate increase of the force of friction with the path of sliding, i.e., the radial force.

In some series of experiments, the friction elements were electrically insulated from the Buckton machine by sheets of mica and were connected to a ballistic galvanometer * which indicated a discharge current with each jerk if the sequence of jerks was sufficiently slow for the mirror of the galvanometer to return to its zero position between two consecutive jerks. Otherwise the galvanometer would exhibit a constant deflection while jerky motion proceeded.

§ 5. RESULTS

Jerky motion and discharge currents. Characteristic results were obtained when a steel pin and ring were assembled with, for instance, butyl acetate, and when a similar pair of friction elements was coated with a machine oil, heated in a furnace with air circulation to 193° c. for, say, one to two hours and then assembled. In the former case, relative motion proceeded by a series of large jerks (figure 3), which increased with the nominal area of contact to reach a maximum value of 0·48 cm. in the fully entered condition. The envelope of the jerks is a trapezoid, the measured gradient being $2{\cdot}55\times10^3$ kgm. wt./cm. and the recorded maximum value of the force of friction $F_{\max}=13{\cdot}6\times10^3$kgm. wt. Since $l_h=5{\cdot}334$ cm., one would calculate $F_{\max}/l_h=2{\cdot}55\times10^3$ kgm.wt./cm., in accordance with Amontons' law, which applies only to sliding that proceeds without tearing of the surfaces. Indeed, the dismantled surfaces did not show any sign of tearing in spite of the heavy jerks which had been observed throughout the process of assembling and dismantling of the elements. The behaviour of the other pair was quite different. No jerky motion was observed with it (figure 4); the force of friction increased at a larger rate than the nominal area of contact; the elements seized, i.e. the tangential force required to continue relative motion in the fully entered condition after a path of sliding of 6·3 cm. had been completed, was beyond the capacity of the machine. Inspection of the surfaces after the hollow element had been cut open, and thus separated from the solid one, showed that

* The authors are indebted to Mr. F. Record, of the Technical College of Derby, for the loan of a ballistic galvanometer.

they had both suffered severe tearing* during a process of smooth continuous sliding. It is obvious from these two examples that tearing is not the result of jerky motion. On the contrary, a large number of experiments have shown that very heavy jerking without an appreciable deviation from Amontons' law is a certain indication that the surfaces are not severely torn during sliding at a small average value of the velocity.

When the pins and rings were electrically insulated from the Buckton machine and connected to a ballistic galvanometer, discharge currents were observed when jerking took place. In accordance with the view that the galvanometer deflections indicated the current which flowed when the contact potential reached the breakdown value of the boundary layer and ceased when the potential had fallen to a smaller value for which the resistance of the boundary layer was practically infinite, an appreciably larger galvanometer deflection was observed at the

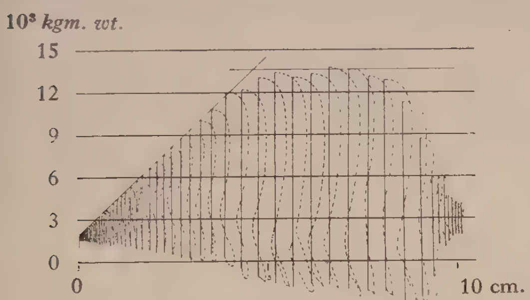

Figure 3.

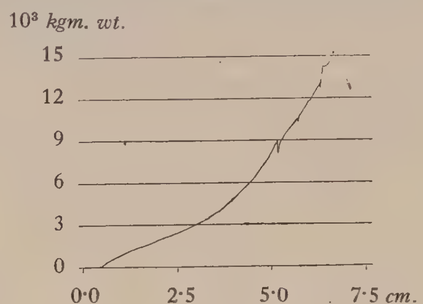

Figure 4.

Figure 3. Jerky motion when a steel pin and ring were assembled slowly and without tearing The frictional force (plotted on the ordinate) increased in proportion to the radial force (Amontons' law). The horizontal distance between the continuous almost-vertical lines indicates the magnitude of slipping. The dotted parts of the record are due to violent vibrations of the recording gear immediately after sudden slipping of the pin into the ring.

Figure 4. Smooth relative motion and deviation from Amontons' law when relative motion proceeded by severe tearing. The small kinks on the curve were obtained when the propelling operation was suspended.

final jerk which made the pin drop out of the ring, because the complete separation of the two friction elements must lead to a *complete* discharge of their surfaces through the galvanometer to which they were connected. At a rate of propulsion of $2{\cdot}67 \times 10^{-3}$ cm./sec., the galvanometer deflection per jerk was observed to be 1·5 mm.† when a brass pin was assembled with a steel ring with Regelax medicinal paraffin-oil (Boots) as lubricant; the maximum value of the force of friction was $9{\cdot}5 \times 10^3$ kgm. wt., and the maximum jerk was 0·38 cm., both values being in close agreement with the results obtained with a steel pin and Regelax medicinal paraffin-oil as lubricant (see table 1). An increase in the rate of propulsion did not increase the galvanometer deflection, as was found when the Buckton machine was operated at a rate of $2{\cdot}67 \times 10^{-2}$ cm./sec., and the recorded maximum value

* See *Engineering*, 1940, **150**, 236.

† This deflection was known from calibration to correspond to 4×10^9 coulombs.

of the force of friction of the solid element of brass dropped to $7 \cdot 82 \times 10^3$ kgm. wt. and the maximum jerk to 0·25 cm. At a still higher rate of propulsion, the discharge periods followed each other so frequently that the galvanometer assumed a steady deflection of, for instance, 2 mm. at $5 \cdot 64 \times 10^{-2}$ cm./sec., 3·5 mm. at $8 \cdot 75 \times 10^{-2}$ cm./sec., and 4·5 mm. at $1 \cdot 137 \times 10^{-1}$ cm./sec. In the last case a deflection by a further 11·5 mm. was observed at the instant of completed dismantling of the friction elements.

It is important to note also that a periodic discharge current was observed to accompany jerky motion when *both* the pin and ring were made of steel. In these particular experiments, Regelax medicinal paraffin-oil had been used as lubricant. The galvanometer deflection per jerk was 0·5 mm., i.e. smaller than with a brass pin, though of the same order of magnitude. This smaller value was probably due to the circumstance that these particular steel friction elements were defective, so that the maximum value of the force of friction was $7 \cdot 7 \times 10^3$ kgm. wt. instead of between $9 \cdot 8 \times 10^3$ kgm. wt. and $10 \cdot 2 \times 10^3$ kgm. wt., and the maximum jerk was 0·26 cm. instead of 0·41 cm. with undamaged steel elements (table 1). These " defective " steel elements had been used previously eight times, and had acquired one bad score during the eighth series of experiments. A shortage of specimens made it necessary to recondition this pair by filing over the scored portions of the surfaces and to use them in the " defective " condition. Thus, since both the friction and the jerks were smaller than with undamaged surfaces, the galvanometer deflection per jerk would also be expected to be smaller. Anyhow, the essential point is that such deflections were clearly observed when both the pin and ring were made of steel.

Further evidence of the galvanometer deflection as an indication of a discharge current was obtained at the high rates of propulsion when the magnitude of the steady deflection varied with the nominal area of contact between the friction elements. For the reason mentioned above (that the contact potential would vary in the assembled condition between the breakdown value and a slightly smaller value for which the resistance of the boundary layer was practically infinite, and between the extinction value and zero when separation of the surfaces took place), the steady deflections observed at the high rates of propulsion were largest during the stage of dismantling and smallest in the fully entered condition. At a rate of propulsion of $8 \cdot 75 \times 10^{-2}$ cm./sec., for instance, a deflection of 3 mm. was observed in the assembling stage, a deflection of 2·5 mm. in the fully entered condition, and a deflection of 3·5 mm. during dismantling. The corresponding observations at a rate of propulsion of $1 \cdot 137 \times 10^{-1}$ cm./sec. were 4 mm., 3 mm., and 4·5 mm. respectively.

Variation of the dielectric breakdown strength of the lubricant

A brass pin and a steel ring were assembled once with Regelax medicinal paraffin-oil, and another time with a mixture of 50 % rape oil and 50 % Russian tallow. The rate of propulsion was $2 \cdot 67 \times 10^{-3}$ cm./sec. in both cases. In the

Figure 5. Damage to the surfaces of the pin and ring after smooth relative motion. The assembly was made with pure caprylic acid which severely attacked the steel, as the top surface of the ring shows.

Figure 6. Severe tearing of the pin and ring after assembly with a mixture of two parts of Regelax medicinal paraffin-oil with one part of octyl alcohol, which attacked the steel, as the top surface of the ring indicates. The blobs of metal which can be seen on the surface of the pin were not welded on and could easily be removed.

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Table 1

Lubricant	Voltage		Interference (10^{-3} cm.)	Rate of propulsion (10^{-3} cm./sec.)	F_{max} (10^3 kgm. wt.) (observed)	$\mu_{max} = F_{max}/P_r$ (computed)	Max. jerk (cm.) (observed)
	Held for 1 min. (kilovolts)	Breakdown (kilovolts)					
Regelax	40	45 (instantaneous)	2·5	2·54	10·1	0·32	0·4
,,	40	45 (instantaneous)	2·5	2·54	9·8	0·31	0·42
,,	40	45 (instantaneous)	2·5	2·54	10·2	0·32	0·41
Castrol XL	50	50 (almost instantaneous)	2·5	2·54	8·7	0·28	0·38
Soft soap	—	—	2·5	2·54	7·5	0·24	0·25
Regelax + 1 % caprylic acid	15	16 (after 15 sec.)	2·5	2·54	6·5	0·21	0·18
Regelax + 5 % technical oleic acid	—	—	2·8	2·67	6·51	0·19	0·15
Castor oil	—	—	2·8	2·67	6·18	0·18	0·20
Raw linseed oil	—	—	2·8	2·67	5·62	0·16	0·05
Rape oil	30	35 (after 40 sec.)	2·8	2·67	5·21	0·15	0·06
Tallow and rape oil	25	30 (after 17 sec.)	2·5	2·67	5·43	0·17	0·14

latter case, both the force of friction and the magnitude of the jerks were smaller, and the galvanometer deflection per jerk was 0·25 mm. instead of 1·5 mm. with Regelax. At a rate of propulsion of $1\cdot137\times10^{-1}$ cm./sec., the galvanometer showed a steady deflection of 4 mm. when the mixture of tallow and rape oil was used as lubricant, and a large increase of this deflection was observed when the two friction elements parted contact.

The friction results obtained with steel elements and various lubricants were compared with the dielectric breakdown strengths of these materials in bulk (table 1).

Very similar friction results had been obtained with proprietary samples of Regelax medicinal paraffin-oil and Castrol XL motor oil (Wakefield) before their dielectric breakdown strengths were measured. A different sample of Castrol XL which had been stored in the works was sent to the chemical laboratory,* where it was found that 11 kilovolts were held for one minute (transient sparking) and that instantaneous breakdown occurred at 15 kilovolts. The chemists, who did not know of the friction results, remarked in their report that the presence of a trace of water in this oil was suspected. They were then supplied with a proprietary sample of Castrol XL and found 50 kilovolts, a result very close to the one obtained with Regelax, and in agreement with the observation that both these lubricants had given similar friction results.

Table 2

F_{max} (10^3 kgm. wt.) (observed)		$\mu_{max}=F_{max}/P_r$ (computed)		Max. jerk (cm.) (observed)	
Castrol High-Press		Castrol High-Press		Castrol High-Press	
Unfiltered	Four times filtered	Unfiltered	Four times filtered	Unfiltered	Four times filtered
5·10		0·15		0·08	
	6·27		0·18		0·175
	5·7		$0\cdot1_7$		0·09
5·7		$0\cdot1_7$		0·07	
6·0		$0\cdot1_7$		0·09	
	6·56		0·19		0·12

The dielectric properties of oils can as a rule be slightly improved by filtration, and the claim has been made recently by du Noüy (1940) that the lubricating properties of, for instance, a good motor oil are diminished when it has been filtered four times through paper. Experiments with Castrol High-Press lubricating oil (Wakefield), both in the "as received" condition and after four filtrations through paper showed larger values both of the force of friction and of the jerks in the latter case (table 2). All but the first of the results summarized

* The authors express their thanks to Mr. E. A. Coakill for the measurement of the dielectric breakdown strengths in accordance with the B.S.I. specification for Insulating Oils for Electrical Purposes (No. 148, 1933).

in table 2 were obtained with the same pin and ring; the interference was $2 \cdot 8 \times 10^{-3}$ cm. throughout.

Sir William Hardy, as is well known, deduced from experiments with various members of a chemical series among the substances used as lubricants that the friction decreased with increasing molecular weight in any given series. This relationship was found to hold for series of acids, alcohols and paraffins. For substances of the same molecular weight but in different series the friction was found to be dependent upon the chemical structure of the molecule. Fogg (1940) measured static friction with a modified form of Deeley machine, using

Table 3

Lubricant	Molecular weight	F_{max} (10^3 kgm. wt.) (observed)	$\mu_{max} = F_{max}/P_r$ (computed)	Max. jerk (cm.) (observed)
Methyl formate	60	13·0	0·38	—
,, acetate	74	11·8	0·34	0·09
,, propionate	88	8·9	0·26	0·25
,, butyrate	102	8·0	0·23	0·18
,, valerianate	116	8·7	0·25	$0{\cdot}22_5$
Ethyl formate	74	13·3	$0{\cdot}3_9$	0·3
,, acetate	88	12·6	$0{\cdot}3_7$	0·45
,, propionate	102	9·83	0·28	$0{\cdot}32_5$
,, lactate	118	9·54	0·28	$0{\cdot}27_5$
,, iso-valerate	130	9·17	$0{\cdot}2_7$	$0{\cdot}27_5$
,, oenanthate	158	7·8	0·23	0·10
,, capryllate	172	6·17	0·18	0·10
,, stearate	312	5·7	0·17	$0{\cdot}07_5$

as lubricants methyl and ethyl esters which by virtue of their chemical inactivity were suitable as regards freedom from surface corrosion. He noticed a maximum value of the coefficient of static friction at molecular weights of about 88 and 120 respectively, and suggested an explanation of this phenomenon by making certain assumptions regarding the structure and method of attachment of the molecules to the surface. In the present experiments with pins and rings both made of steel (interference: $2 \cdot 8 \times 10^{-3}$ cm.), and methyl and ethyl esters as lubricants, at a rate of propulsion of $2 \cdot 67 \times 10^{-3}$ cm./sec., the maximum values of the force of friction were found to decrease in both series with increasing molecular weight (table 3), but when esters with the same acid radical and various alcohol radicals were compared (table 4), the rule did not apply that the friction decreased with increasing molecular weight.

The electrical conductivities at 25° c. of methyl acetate ($\kappa = 34 \times 10^{-7}$) and of ethyl acetate ($\kappa < 1 \times 10^{-9}$) are recorded in the literature (*International Critical Tables*, 1929, vol. 6, p. 143). If the dielectric breakdown strength can be assumed to be inversely proportional to the conductivities, the friction ought to be larger with ethyl acetate as lubricant in accordance with the experimental results

(table 3). From some relative values of the electrical conductivities of ethyl esters at room temperature (*op. cit.* p. 145, figure 2) one might expect the friction to be larger with ethyl acetate as lubricant than with ethyl propionate (table 3). However, the comparison of all the experimental results obtained with esters with the recorded relative values of the electrical conductivities would also suggest some inconsistencies. The possibility was not ruled out that the

Table 4

Lubricant	Molecular weight	F_{max} (10^3 kgm. wt.) (observed)	$\mu_{max} = F_{max}/P_r$ (computed)	Max. jerk (cm.) (observed)
Methyl acetate	74	11·8	0·34	0·09
Ethyl ,,	88	12·6	$0{\cdot}3_7$	0·45
Propyl ,,	102	11·2	0·32	0·36
Butyl ,,	116	13·6	0·39	0·48
Iso-propyl ,,	118	13·3	$0{\cdot}3_9$	0·46
Amyl ,,	130	12·0	0·35	0·4
Propyl phthalate	250	11·2	0·32	0·36
Butyl ,,	278	11·3	0·33	0·45
Amyl ,,	306	11·6	0·34	$0{\cdot}42_5$

Table 5

Lubricant	Interference (cm.)	Rate of propulsion (10^{-3} cm./sec.)	F_{max} (10^3 kgm. wt.) (observed)	$\mu_{max} = F_{max}/P_r$ (computed)	Max. jerk (cm.) (observed)
Machine oil	$2{\cdot}03 \times 10^{-3}$	2·54	5	0·2	0·2
		5·08	—	—	0·2
		10·2	4	0·16	$0{\cdot}12_5$
		31·7	2·5	0·1	$0{\cdot}02_5$
Machine oil (heated to 200° c. for 30 min. while air bubbled through)	$2{\cdot}54 \times 10^{-3}$	2·54	5	0·16	0·12
		5·08	4·7	0·15	0·1
		10·2	4·38	0·14	0·1
		31·7	3	0·1	
Regelax	$2{\cdot}54 \times 10^{-3}$*	2·67	9·54	0·37	0·35
		26·7	7·82	0·30	0·25
		56·4	6·32	0·24	0·17
		87·5	6·0	0·23	$0{\cdot}17_5$
		113·7	5·16	0·2	$0{\cdot}07_5$
Regelax	$7{\cdot}62 \times 10^{-3}$†	2·54	5·5	$0{\cdot}1_8$	0·16
		5·08	5	$0{\cdot}1_6$	0·09
		10·2	4·54	0·14	0·07

* The solid element was made of brass.

† The length of the ground surface of the ring was 1·778 cm. The ring became stretched in the course of the experiment.

volatility of the lower members of the series might be responsible for these. It will be recalled that the ground surfaces of a pin and ring were flushed with an ester immediately before they were assembled, and that in the case of volatile liquids a dropping arrangement maintained a surplus of lubricant around the leading edge. The possibility that in these cases the ground surface of the ring dried up while the pin entered cannot be excluded.

Variation of the rate of energy supply

It was mentioned above that the rate of energy supply to a device with a characteristic exhibiting a non-linear falling section and a minimum would decide whether the functioning of the device was unstable or stable. In the present case, the velocity at which the Buckton machine is operated, i.e. the rate of propulsion of one friction element, represents the rate of energy supply. As this increases from zero, the oscillations, i.e. the jerks, must decrease in the unstable region, and must give way to continuous motion (smooth sliding) at a sufficiently high velocity for stability to be ensured. Table 5 illustrates the decrease of both the maximum value of the force of friction and the jerks with increasing rate of propulsion.

Oxidation and tearing

The electrostatic component of the force of sliding friction decreases as the dielectric breakdown strength of the boundary film is reduced, and the jerks become correspondingly smaller. This is the case, for instance, when one per cent of caprylic acid is added to Regelax medicinal paraffin-oil and sliding proceeds without appreciable damage to the metal surfaces, so that Amontons' law is obeyed. No jerks were observed when pure caprylic acid was used as lubricant for friction elements of steel. But in this case the metal surfaces (interference: $2{\cdot}54 \times 10^{-3}$ cm.) suffered severe tearing (figure 5), and the force of friction reached 7×10^3 kgm. wt. (rate of propulsion: $2{\cdot}67 \times 10^{-2}$ cm./sec.) in the fully entered condition of the solid element. The caprylic acid was found to attack the steel. Tearing and deviations from Amontons' law were always observed when either the material used as lubricant appreciably attacked the metal surfaces or when these had become discoloured with corrosion before the lubricant was applied. For instance, steel friction elements which had been standing without a coat of wool-fat after the grinding operation had become slightly discoloured with corrosion before they were assembled with tallow and rape oil. Very few small jerks occurred, and the surfaces suffered severe tearing. Standard soap solution, and a mixture of two parts of Regelax medicinal paraffin-oil with one part of octyl alcohol when used as lubricants also gave rise to severe tearing (see figure 6). The friction records exhibited appreciable deviations from Amontons' law. The plucked out particles were easily levered off the surfaces; they were *not* blobs of welded-on metal.

§6. THE LIMITING VALUE OF THE FORCE OF "STATIC FRICTION"

As a result of static friction, the elastic force increases as one friction element is moved forward at a uniform velocity while the other one is elastically restrained. When the elastic force reaches the limiting value of the force of static friction, the slider must begin to move, because for small velocities of sliding the force of friction is smaller than the limiting value of static friction, and falls off with an increase in velocity in those cases in which jerky motion is observed. The electrostatic interpretation of jerking would require that the cycles of sticking and slipping should be, in fact, cycles of slow charging and rapid discharging, i.e. cycles of slow sliding and rapid slipping. This would mean that the limiting value of static friction was reached by elastic deformation, together with a slight actual displacement at the contact area of the two bodies, which would be accompanied by increased charge separation until the contact potential reached the breakdown value. The slight displacement preceding slipping was too small to be noticed with the sensitivity of the measuring equipment fitted to the testing machine. The elastic distortion of the machine and the specimens was measured in a separate experiment, when a steel plate prevented the pin from entering the ring, with the result that it amounted to $7{\cdot}5 \times 10^{-3}$ cm. for every 1×10^{3} kgm. wt.

In addition to the above-described experiments, which support the view that the electrostatic component of the force of sliding friction can assume appreciable proportions if the charge separation increases as sliding proceeds, the contribution of the electrostatic force to the radial force can also be estimated. Helmholtz assumed that when two dissimilar surfaces are in contact, an electrical double layer of about 10^{-8} cm. in thickness exists between them. The difference between the breakdown potential and the truly static value might be assumed to be one-tenth of a volt, so that a radial electrostatic force of 4×10^{3} kgm. wt. in the fully entered condition would be computed from

$$P_{el} = \frac{\pi \times d \times l_h}{8\pi(10^{-8})^2} \frac{(10^{-1})^2}{8{\cdot}83 \times 10^{10}}.$$

If the electrostatic interpretation of the friction characteristic with a non-linear falling section and a minimum is correct, no jerking can be expected when two *naked* metal surfaces are in frictional contact, because in this particular case the charge separation does not increase as sliding proceeds, since the positions of the charges are *not* fixed in the metal surfaces. It is a matter of considerable experimental difficulty to work with naked metal surfaces, but under conditions of severe tearing the re-contamination of the freshly created contact area from the surrounding atmosphere may require a short time. Bowden and Leben (1939) reported that an elastically fixed platinum slider in frictional contact with silver in the air of the laboratory exhibited slight jerking at the beginning of the run (the friction element of silver was moved forward at a uniform velocity of a few thousandths of a centimetre per second). "The friction was low", the coefficient being about 0·6 (Bowden, Leben and Tabor, 1939), and a smooth

groove was cut in the silver. After relative motion had proceeded over a short distance, the behaviour of the motion changed, jerking disappeared, and " the friction rose to a high value ", the coefficient fluctuating now between about $1{\cdot}2_5$ and $1{\cdot}3_5$. A close examination showed that a small blob of silver had become attached to the platinum surface, and it would appear that the friction characteristic of the naked metal-to-metal contact created by plucking the blob of silver from the silver plate did not allow of jerking, until the inevitable re-contamination by matter adsorbed and condensed from the air of the laboratory had been completed.

§ 7. DISCUSSION

A different interpretation of sliding which proceeded by a series of jerks was proposed by Bowden and Leben, who suggested that the friction behaviour should depend upon the relative physical properties of the metals, particularly on the melting point rather than upon the properties of the boundary layer. Friction should be due to a welding together of the metals at the local points of contact, and the metallic junctions, when broken, should distort the metal " to a considerable depth ". In their detailed discussion of Bowden and Leben's suggestion, Haykin and his collaborators pointed out that local melting—if it does take place—would be the *result*, and not the *cause* of jerky motion. But even as the *result* of short-lived slipping, no large temperature rise can be expected, and it is well to bear in mind that at every stage the temperature of the surface irregularities is determined by the equilibrium between the amount of heat generated and that dissipated, and that the dissipation of heat by conduction, radiation and convection also increases with the temperature, so that, as Bowden and Ridler (1936) have shown experimentally, *large* values both of the force of friction and of the speed of sliding are required to achieve an appreciable temperature rise (Schnurmann, 1939). The temperature rise under the conditions of Bowden and Leben's experiments can be estimated to be about 1° C. if each jerk is completed in one-thousandth of a second. For this estimate, the results obtained by these authors with silver sliding on a steel plate which was moved forward at a velocity of 6×10^{-3} cm./sec. were used. The silver slider was assumed to be a sphere of $0{\cdot}3$ cm. diameter, and the area of contact between silver and steel was estimated from the Brinell hardness of silver.

The present experiments show clearly that jerking is due to the characteristics of sliding, and not to scoring. When the conditions are such that the electrostatic component of the force of sliding friction is appreciable, and varies as sliding proceeds, jerky motion must arise. This explains Kaidanovsky and Haykin's observation that the friction characteristic of a pair of friction elements of wood and a metal has a similar form to that of two dissimilar metals in the air of the laboratory.

One experiment was made with the intention of reducing the jerks by passing an electric current through the frictional contact. $F_{\max} = 8{\cdot}5 \times 10^3$ kgm. wt. was

found when a steel pin and ring with an interference of $2\cdot8\times10^{-3}$ cm. were assembled with Regelax medicinal paraffin-oil at a rate of propulsion of $2\cdot67\times10^{-3}$ cm./sec., and currents up to 2 amp. passed the contact. The maximum jerk was $0\cdot23$ cm. instead of $0\cdot41$ cm. (table 1) without the current. Much higher current densities would be required for obtaining smooth sliding due to dielectric breakdown of the entire boundary layer.

§ 8. ACKNOWLEDGEMENT

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REFERENCES

Bowden, F. P. and Leben, L., 1939. *Proc. Roy. Soc.* A, **169**, 371.
Bowden, F. P., Leben, L. and Tabor, D., 1939. *Engineer*, **168**, 214.
Bowden, F. P. and Ridler, K. E. W., 1936. *Proc. Roy. Soc.* A, **154**, 640.
Fogg, A., 1940. *Proc. Phys. Soc.* **52**, 239.
Haykin, S., Lissovsky, L. and Solomonovich, A., 1940. *J. Phys. U.S.S.R.* **2**, 253.
Kaidanovsky, N. L. and Haykin, S., 1933. *J. Techn. Phys.* **3**, 91.
du Noüy, P. L., 1940. *C.R. Acad. Sci., Paris*, **209**, 101.
Schnurmann, R., 1939. *Engineer*, **168**, 278.
Schnurmann, R., 1941. *Proc. Phys. Soc.* **53**, 547.

THE ABSOLUTE MEASUREMENT OF ELECTRICAL RESISTANCE BY A METHOD USING THE AVERAGE ELECTROMOTIVE FORCE OF A COMMUTATING GENERATOR*

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ABSTRACT. An electrical resistance of approximately 1 ohm is measured absolutely with rapidity and precision in terms of a mutual inductance and a frequency, by balancing the e.m.f. across it when conveying direct electric current, against the average e.m.f. of a commutating generator, the field coils of which are traversed by the same current.

By a new method, a remarkable " flat " of mutual inductance is obtained between the rotor and the field coils over a range of 20° of arc in the neighbourhood of the commutations. Further, over an adjustable portion of this range, covering the actual breaks at commutations, the rotor together with additional adjustable resistance is short-circuited,

* A preliminary investigation of this method has been described by one of us (E. G. B.) in Part II of a Thesis for the Ph.D. degree of the University of London.

thus enabling the e.m.f. across the resistor to suffer no interruption in its supply to the detector and allowing it to supply the extra quantity of electricity absorbed by the rotor, in virtue of its self-inductance, on establishing the current through it at each commutation. In this way the two main errors associated with break and self-inductance are set against one another, and compensation is readily effected by the adjustment of a resistance in a preliminary experimental test carried out under conditions in which these two effects preponderate.

The rotor has a constant speed of 12·5 revolutions per second, and the maximum mutual inductance of some 20,000 μH. permits of rapid and fine adjustment over a range of 400 μH. The balance is sensitive to 0·1 μH. and an accuracy of a few parts in a hundred thousand has been obtained.

§ 1. INTRODUCTION

THE method of measuring an electrical resistance absolutely by balancing the e.m.f. across it when conveying direct electric current, against the average e.m.f. across a commutating generator, the field coils of which are traversed by the same current, was suggested by Rosa (1909). Although Rosa proposed a form of apparatus and a method of procedure, difficulties were encountered and no experimental data, derived from any form of this method, have hitherto been published (Curtis, 1937).

The method is attractive on account of the high sensitivity, due to the relatively high e.m.f. involved, and because of the simplicity of the basic expression for R, the resistor to be measured, viz.,

$$R = 4n \,.\, M_0, \qquad \ldots\ldots(1)$$

where n is the frequency of revolution of the generator and M_0 is the maximum mutual inductance between the rotor and the field coils.

On the other hand, before the method can attain precision value, three sources of error associated with the break at commutation must be overcome. The first of these, due to the interruption of the e.m.f. of the generator, is adequately rendered negligible by arranging that the mutual inductance between the rotor and the field coils is very flat around the maximum value, so that over the range of commutation the e.m.f. is sensibly zero. The remarkable nature of the "flat", described in this paper, and the additional safeguard associated with the short-circuit device and the method of measuring the effective mutual inductance do away with this trouble completely. A much more serious error, however, may be expected to arise from the interruption through the detector of the steady e.m.f. drawn off the resistor under test, while a third source of error which might be considerable is associated with the effect of self-inductance of the rotor, through which a current is re-established at each commutation.

In order to overcome the major difficulties, Rosa suggested the use of a differential galvanometer as the detector, one coil of which is in uninterrupted connection with the resistor, while other coils are connected suitably with his

proposed two-phase composite rotor. This use of a differential galvanometer, however, which permits quantities of electricity, whose aggregate values over a period of revolution are large, to flow through the various coils, adds considerable complication to the method and prevents the realization of a simple null balance independent of the resistance of the detector circuit.

In the method here described, the e.m.f. across the resistor is directly balanced for zero aggregate quantity of electricity through the galvanometer (over a period of revolution and very approximately over half a period) against the average e.m.f. of the commutating generator. By a new and simple method, the rotor is made to have an extraordinarily flat maximum of mutual inductance with the field coils over some 20° of arc in the neighbourhood of commutation. Further, over an adjustable portion of this range of some 5° to 8°, completely covering the actual commutations, the rotor, together with some small resistance, is short-circuited out of the detector loop, thus enabling the e.m.f. across the resistor to function through the detector without interruption and to supply the extra quantity of electricity absorbed by the rotor at each commutation on account of its self-inductance. In this way, the two main errors arising from the break of the resistor and from the self-inductance of the rotor may be set against one another and compensation is readily effected by experimental test under conditions in which these effects are magnified.

A form of apparatus has been designed for the specially accurate measurement of resistances of approximately 1 ohm, thus enabling the international ohm to be measured in C.G.S. units. For this purpose, the rotor is driven by a synchronized motor at a constant speed of 12·5 rev. per sec. which may be checked by a 1000-cycle note, and the mutual inductance of some 20,000 μH. between the rotor and the field coils at the boundaries of short circuit is varied for final balance on a high-sensitivity galvanometer. The balance, sensitive to 0·1 μH., is obtained and confirmed in a few minutes; the rotor is then stopped and set by an electrical contact test at each in turn of the four limiting positions of short-circuit, where the mutual inductance is measured to 0·1 μH. under the same direct primary current as that used throughout the main experiment, by balance against a 10 mH. certified standard together with a trifle over 10 mH. on a Campbell mutual inductometer which can readily be checked against the standard. The sum of the four readings represents the flux change per unit current per revolution and replaces $4M_0$ in the basic equation (1), which is shown below to be accurate under the experimental conditions to about three parts in a million.

§ 2. DESIGN OF A SUITABLE GENERATOR

Attainment of a flat maximum of mutual inductance between a rotor and twin field coils

Rosa (1909) proposed the use of twin field coils set somewhat further apart than the Helmholtz distance of separation, with a rotor of large diameter set symmetrically between them. A more convenient and efficient arrangement

consists of two twin field coils (A and B, figure 1) separated by any small distance and two relatively small twin-rotor coils, C and D, with their planes parallel and displaced by a distance d from the axis of rotation O. The distance d is

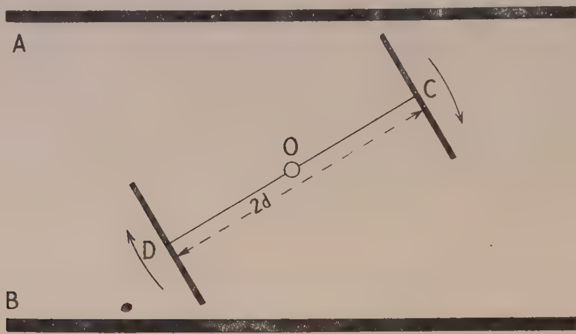

Figure 1. Form of generator.

approximately equal to half the radius of the field coils, and by slight adjustment of this distance the maximum "flat" of mutual inductance is readily obtained. The actual rotor is seen in figure 8. The degree of "flat" of mutual inductance

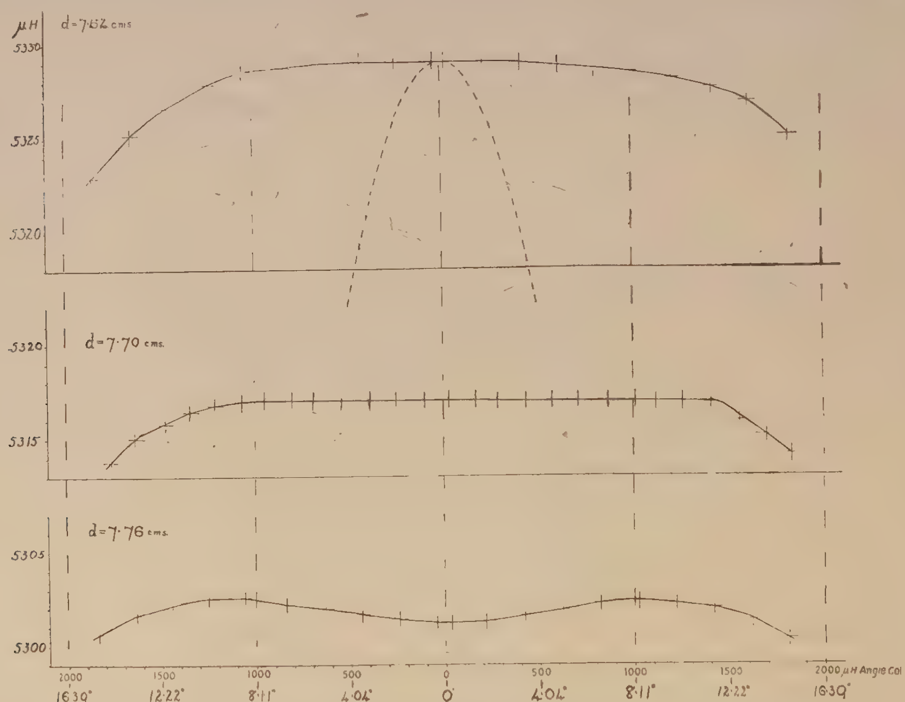

Figure 2. The "flat" of maximum mutual inductance between a rotor coil and field coils for three different displacements of the rotor coil from the axis of rotation. By contrast, the sine law is shown by the dotted curve.

which may be obtained around the maximum is illustrated by the curves of figure 2. These were obtained with our twin field coils of 16·4 cm. mean radius,

and one of the coils C of the rotor used in the preliminary investigation. This coil, of 334 turns of d.s.c. copper wire of S.W.G. 26 was wound on a former having a channel 1·2 cm. wide and an internal diameter of 7·6 cm. These curves show the variation with angular displacement of the mutual inductance between this rotor coil and the field coils at three slightly different distances d of displacement of C from the axis of rotation. The contrast from the sine law, which holds approximately when d is zero, is shown for the same maximum by the dotted curve. In the position of maximum flat, the extreme variation of mutual inductance is only 0·2 μH. in 5317 μH. over a range of 20° of arc, and a setting of the rotor coil between this position and that for the lowest curve is ideal. The coil D is similarly set and then joined in series and in conjunction with C. The angular displacements, though not required accurately, were most conveniently measured by an angle coil necessary for the wave-form determinations below (Llewellyn, 1934; Nettleton and Balls, 1935; Balls, 1938).

Wave form of the e.m.f. of the generator and of the currents through the detector and rotor at null balance

With the aid of an angle coil obeying the sine law, inserted in the hole h of the oak bed (figure 8) of the rotor used in the preliminary investigation, the mutual inductance M between the rotor and the field coils was explored at various displacements θ from the conjugate position. The variation of M with θ enabled the corresponding values of $dM/d\theta$, proportional to the generated e.m.f. under constant speed of revolution, to be deduced and plotted against θ as a wave form.

In the actual experimental measurement of resistance, a balance for zero aggregate quantity of electricity through the detector must be obtained from the two opposing e.m.f.s. shown in figure 3, the one, a steady e.m.f. drawn off the resistor under test (ignoring the effects of break and self-inductance), and the other, the variable e.m.f. of the generator, having zero value in the neighbourhood of commutation.

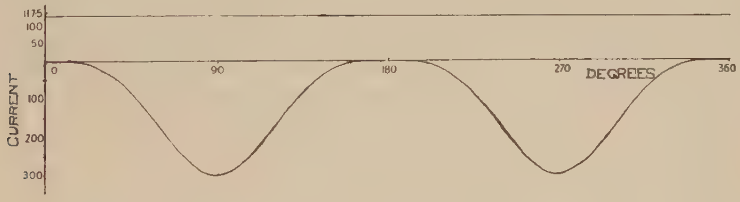

Figure 3. The steady e.m.f. opposed by a variable e.m.f. from the generator.

The resultant wave form of the current through the detector at balance is shown in figure 4, and has a period equal to half the time of revolution of the rotor; it is important in the complete theory of the method given below. Over a range of about 50°·5 on each side of the maximum of mutual inductance, a current flows through the detector in a positive direction, the flat maximum

value, c_0, being equal to that through the detector when the rotor is at rest; over the range of 39°·5 on each side of the zero of mutual inductance and maximum e.m.f. of the generator, an equal quantity of electricity flows through the detector in the negative direction.

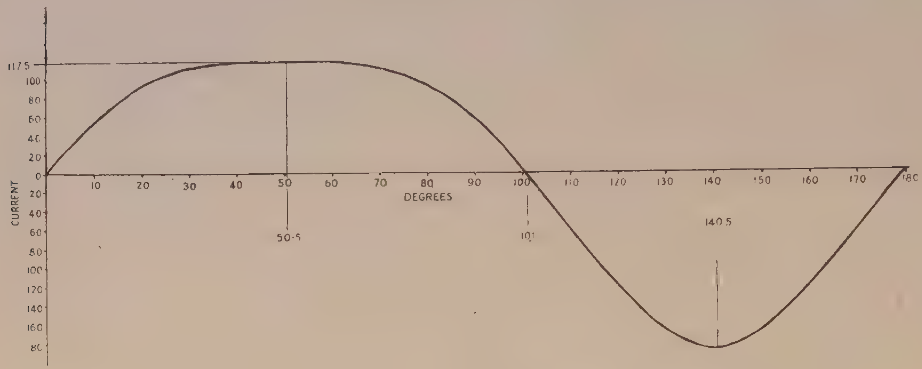

Figure 4. The wave form of the current through the detector at balance.

The wave form of the current through the rotor at balance for zero aggregate quantity of electricity through the detector is shown in figure 5, and has a period

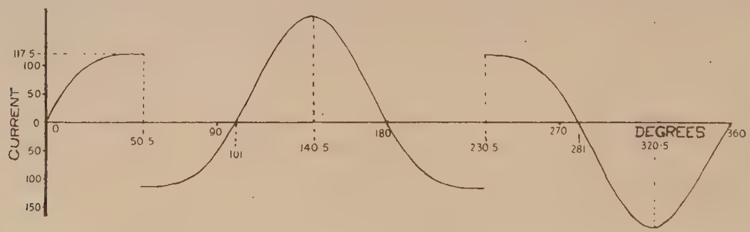

Figure 5. The wave form, at balance, of the current through the rotor.

equal to the time of revolution of the rotor. The current c_0 is reversed through the rotor at each commutation.

The effects of the short circuit covering the commutations and of self-inductance are considered in the next section.

§ 3. THEORY OF THE METHOD

The circuit and fundamental equations

In the arrangement shown in figure 6, the primary circuit consists of a steady voltage E in series with the resistor R to be measured and additional resistance X, which includes the field coils of the generator and rheostats of total self-inductance μ. The secondary circuit drawn off the potential leads of R is of total resistance $R+F$. The resistance F is of at least 8000 ohms, and includes a high-resistance galvanometer G of self-inductance λ, additional non-inductive resistance, and the resistance r, which comprises the rotor of resistance

$r_1 = 17{\cdot}5$ ohms and self-inductance $L = 0{\cdot}0385$ H. and the additional non-inductance resistance r_2 in ohms and tenths up to 20 ohms. The shorting rotor N is adjustable and enables the commutations to be covered over various angles of from 4° to 9° of arc; when operative, it cuts out a resistance $r = r_1 + r_2$ from the secondary circuit.

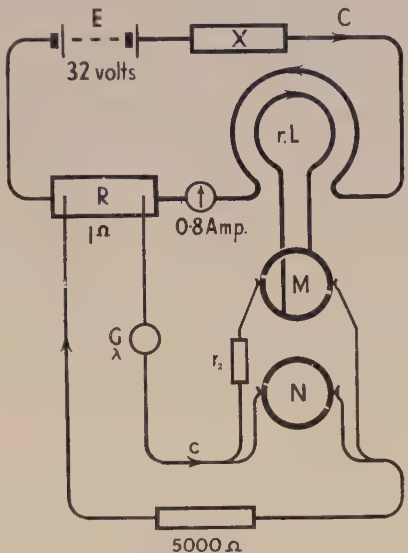

Figure 6. The essential circuit.

Let C and c respectively denote the instantaneous values of the currents in the primary and secondary circuits, and M the instantaneous mutual inductance between them. Then, with the short-circuit inoperative, we have, by applying Kirchhoff's laws to the circuits,

$$CX + (C - c)R = E - d(cM)/dt - \mu \,.\, dC/dt, \qquad \ldots\ldots(2)$$

$$cF - (C - c)R = -d(CM)/dt - (\lambda + L)dc/dt, \qquad \ldots\ldots(3)$$

whence

$$c(FX + FR + XR) = ER - (X + R) \,.\, d(CM)/dt$$
$$- (X + R)(\lambda + L)dc/dt - Rd(cM)/dt - R\mu dC/dt, \qquad \ldots\ldots(4)$$

which may be written in very convenient form, showing the quantity of electricity dQ traversing the detector in the time dt, viz.,

$$dQ = c \,.\, dt = [ER \,.\, dt - (X + R) \,.\, d\phi - R \,.\, d\psi]/Y, \qquad \ldots\ldots(5)$$

where $d\phi$ = flux change in the detector circuit in the time dt,
$d\psi$ = flux change in the primary circuit in the time dt,

and $$Y = FX + FR + XR. \qquad \ldots\ldots(6)$$

Owing to the large " flat " of maximum mutual inductance, no flux changes occur over several degrees surrounding the short circuit and commutation, and

C, c and M are constant. The limiting values of C and c just before and just after short circuit are then given by

$$C_0 = E(F+R)/Y, \qquad \ldots\ldots(7)$$

$$c_0 = ER/Y. \qquad \ldots\ldots(8)$$

Elsewhere, the value of C only differs minutely from the value C_0, but the variation of c at balance is that shown by the wave form of figure 4.

There are various flux changes and transients associated with the commutations and short circuits that cover them, but the most evident effect is the modification of the current c_0 due to the removal of the resistance r from the secondary circuit. On short circuit, the current through the detector, almost instantaneously attains a value c_s given by

$$c_s = ER/Y_s, \qquad \ldots\ldots(9)$$

where

$$Y_s = (F-r)(X+R)+XR; \qquad \ldots\ldots(10)$$

and hence, using (6) and (8),

$$c_s = c_0\left[1 + \frac{r}{F-r+RX/(X+R)}\right]. \qquad \ldots\ldots(11)$$

Now $F+RX/(X+R)$ is the resistance of the detector circuit with the rotor unshunted and with the primary circuit closed as a shunt across R; whence, writing F_0 for $F+RX/(X+R)$, we have, without approximation,

$$c_s = c_0[1+r/(F_0-r)]. \qquad \ldots\ldots(12)$$

The effect of the short circuit on the value of C_0 for the primary circuit will be seen to be insignificant.

Under the ideal conditions of negligible self-inductance of the rotor and negligible times of break and short circuit at commutation, as the rotor revolves with a frequency n and period T, the condition for balance in the detector is readily seen. For in a time $T/2$ the value of M changes from a negative maximum, $-M_0$, just after commutation to a positive maximum, M_0, just before the next commutation. The flux change, ϕ, in the detector circuit due to the sweep in the time $T/2$ is thus $2CM_0$, for there is no flux change due to the commutation or to the self-inductance, λ. The flux change, ψ, in the primary circuit in the time $T/2$, is algebraically zero, because the change $2c_0M_0$ between the commutations is neutralized by the same amount of opposite sign on actual commutation, whilst the self-inductance, μ, is without influence in half a period. Thus the integral of (5) gives

$$Q = \int_0^{T/2} c \,.\, dt = \frac{ER}{Y} \cdot \frac{T}{2} - \frac{X+R}{Y} \,.\, 2C_0M_0, \qquad \ldots\ldots(13)$$

and for zero aggregate quantity of electricity through the detector over any number of half cycles, we have

$$R = 4 \,.\, nM_0 \,.\, C_0(X+R)/E, \qquad \ldots\ldots(14)$$

which, by virtue of (6), (7) and (8), is more usefully expressed

$$R = 4 \,.\, nM_0[1 + c_0R/E] \qquad \ldots\ldots(15)$$

$$R = 4 \,.\, nM_0[1 + R^2/(FX + FR + XR)]. \qquad \ldots\ldots(16)$$

The balance when the rotor is inductive and is short-circuited at commutation

A complete period of rotation is made up of the time $t = t_1 + t_2$ of the two approximately equal short circuits which cover the commutations, and of the two longer intervals of combined duration $(T - t)$.

During the time $(T - t)$, the motional flux changes are $\phi = 4C_0M_0$ in the detector circuit and $\psi = 4c_0M_0$ in the primary circuit, producing flows in the negative direction in the detector; there is no resultant flux change due to any of the self-inductances, μ, λ and L. Thus, from (5) and (8), the quantity of electricity which passes is

$$Q_{T-t} = c_0(T - t) - 4 \,.\, C_0M_0 \,.\, (X + R)/Y - 4 \,.\, c_0M_0R/Y. \quad \ldots\ldots(17)$$

During the time t, the current through the detector is c_s of (11) and (12), and during the initial portions of each short circuit, the flux c_0L through the isolated rotor decays through r, being finally destroyed by the momentary break at commutation. The destruction of this flux is without influence on the detector. On leaving each short circuit, the current c_0 is re-established in the rotor in the contrary direction. Thus, as a result of each short circuit and commutation, a flux change, $\phi = c_0L$, having negative influence, is produced in the detector circuit, while a flux change, $\psi = 2c_0M_0$, having positive effect on the detector, is produced in the primary circuit. A small portion of this last effect on the primary may reach the detector while the secondary circuit is reduced by r, i.e., with Y_s of (10) replacing Y of (6); but the effect of this can be shown to be negligible, not affecting the expression for R by more than a few parts in 10^9. The self-inductance, λ, is without effect on the quantity of electricity conveyed by the current c_s. Thus from (5) and (12) the quantity of electricity which passes the detector due to both short circuits and commutations is

$$Q_t = c_0\left[1 + \frac{r}{F_0 - r}\right]t - \frac{X + R}{Y} \,.\, 2c_0L + \frac{R}{Y} \,.\, 4c_0M_0. \quad \ldots\ldots(18)$$

Adding (17) and (18), we have for the quantity of electricity traversing the detector in a whole period

$$Q_T = c_0T - \frac{X + R}{Y} \,.\, 4C_0M_0 + \frac{c_0rt}{F_0 - r} - \frac{X + R}{Y} \,.\, 2c_0L. \quad \ldots\ldots(19)$$

The condition that the last two terms shall have no resultant effect is

$$t = 2L(1/r - 1/F_0), \quad \ldots\ldots(20)$$

and, if this condition is satisfied, Q_T will be zero when the first two terms are equal, that is, when R has the value given by (14), (15) and (16). Under actual experimental conditions, the correcting term of (16) is of the order of 3 parts in 10^6, so that the basic expression, (1), is sufficiently accurate.

Attainment of balance between the effects of short circuit and self-inductance

The last two terms of (19) have, in actual experiment, an order of magnitude of some 1·2 parts in 10^4 when compared with the equal fundamental terms in T and M_0. By equating these minor terms to an accuracy of 2 % the combined error due to short circuit and self-inductance will be less than 3 parts in 10^6.

This is done by carrying out a compensation test with the arrangement seen in essence in figure 7 below, in which the rotor is switched into a simple circuit

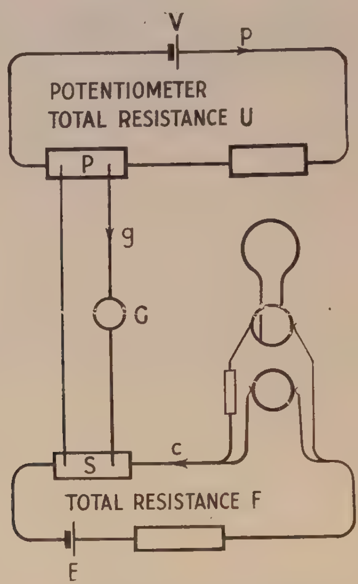

Figure 7. Essence of the compensation test.

of some 1800 ohms, containing a 2-volt accumulator. The e.m.f. of about a volt drawn off half this resistance is balanced on a suitable potentiometer with the rotor at rest and unshunted. The rotor is then set spinning at the standard speed of rotation and the potentiometer balance examined. By adjustment of r_2, the dynamical and statical potentiometer balances are made identical. The potentiometer, under the actual conditions of test, was a measurer of the average current under the dynamical conditions to an accuracy of $1\frac{1}{4}$ %, and the effect of the earth's field, which is neutralized by field coils, has less than 5 % of its sensitivity in the neutralizing test.

If, in this compensation test, F_s is the total resistance of the rotor circuit, $r_s = r_1 + r_2$, the resistance cut out by short circuit, and $\bar{c}$, the statical current balanced off S on the potentiometer, we have, on attaining identity of statical and dynamical balances,

$$\bar{c}\,.\,T = \int_0^T \bar{c}\,dt = \bar{c}T + \bar{c}r_s t/(F_s - r_s) - \bar{c}\,.\,2L/F_s, \qquad \ldots\ldots(21)$$

whence $$t=2L(1/r_s-1/F_s). \qquad \ldots\ldots(22)$$

Now in the main experiment, the effective resistance F_0 of the detector circuit is greater than F_s; the balancing value r_s in the compensation test should accordingly be raised to r, where

$$1/r=1/r_s-1/F_s+1/F_0. \qquad \ldots\ldots(23)$$

Thus, with $r_s=25$ ohms, $F_s=1800$ ohms, $F=8000$ ohms, then $r=25{\cdot}27$ ohms.

The experimental value of r needed for compensation in the main experiment, corresponding to any value of the mean angle θ of the two short circuits, is in good agreement with that calculated, thus:—Let θ_1, θ_2 in degrees of arc be the two angles of short circuit, covered in the times t_1, t_2 during the period T. Then

$$t_1+t_2=t=T(\theta_1+\theta_2)/360=T\theta/180, \qquad \ldots\ldots(24)$$

whence, from (20),

$$2L(1/r-1/F_0)=T\theta/180, \qquad \ldots\ldots(25)$$

and, since F_0 is large, $L=0{\cdot}0385$ H., $1/T=12{\cdot}5$, we have approximately

$$r\theta=173. \qquad \ldots\ldots(26)$$

Experiments have been performed with various values of θ ranging from $5^\circ{\cdot}2$ to $8^\circ{\cdot}65$ and corresponding values of r of from $33{\cdot}5$ ohms to $20{\cdot}2$ ohms.

Use of the potentiometer to measure average current in the special circumstances

Applying Kirchhoff's laws to the three loops of figure 7, and using the notation of the diagram, we have, in general, for the current through the galvanometer when the rotor is unshunted,

$$g=[VP/U-(E-L\dot{c})S/F]/Z, \qquad \ldots\ldots(27)$$

where $$Z=G+P-P^2/U+S-S^2/F, \qquad \ldots\ldots(28)$$

and, on securing static potentiometer balance,

$$PFV=SUE \qquad \ldots\ldots(29)$$

On spinning at standard frequency, F is reduced to $(F-r)$ and Z is reduced to a new value Z' during the times of short circuit of total duration t; a current of value

$$g'=-E\,.\frac{S}{F}\frac{r}{F-r}\,.\,/Z' \qquad \ldots\ldots(30)$$

then flows through the galvanometer.

During each of the intervals between the short circuits, the current through the galvanometer is for the most part zero, but at the beginning of these intervals there is a deficit in c due to the establishment of current in the inductive rotor and a flow of electricity through the galvanometer of quantity:

$$Q_g=ESL/F^2Z. \qquad \ldots\ldots(31)$$

Thus, for balance still to hold under the dynamical conditions at the statical setting of the potentiometer,

$$tg' = 2Q_g, \qquad \ldots\ldots(32)$$

i.e.

$$\frac{t}{2L} \cdot \frac{Fr}{F-r} = \frac{Z'}{Z}, \qquad \ldots\ldots(33)$$

or

$$t = 2L(1/r - 1/F)(1-\alpha), \qquad \ldots\ldots(34)$$

where

$$\alpha = r\,.\,S^2/F(F-r)Z. \qquad \ldots\ldots(35)$$

Hence, if $S = F/2$, we have, approximately

$$\alpha = r/4(P - P^2/U + G + F/4), \qquad \ldots\ldots(36)$$

and with $r = 25$ ohms, $G = 25$ ohms, $P = 50$ ohms, $U = 100$ ohms, $F = 1800$ ohms and $S = 900$ ohms, we have $\alpha = 1/80$. A galvanometer of some 500 ohms would reduce α to less than 1/150 and be otherwise advantageous.

The earth's vertical field is neutralized in this test as in the main experiment, and the dynamical balance is taken for both directions of a reversing key at the rotor brush terminals. It is advantageous, however, that any small average e.m.f. dE, due to imperfect neutralization, should have smaller effect than in the test for neutralization. The current through G due to dE in the potentiometer test is

$$g = dE\,.\,S/FZ. \qquad \ldots\ldots(37)$$

With $S = F/2 = 900$ ohms, this is only some 5 % of the effect when $S = F$, and can be shown to be about $4\frac{1}{2}$ % of the effect in the direct test for neutralization.

§ 4. THE EXPERIMENTAL ARRANGEMENT

The general form and lay-out of the main apparatus will be gathered from figures 8, 9 and 10. The twin field coils A, B, (figure 8) were constructed of dexonite. The channels of radial depth and axial breadth, 3·6 cm., were each filled with 504 turns of d.s.c. copper wire of S.W.G. 16, the mean diameter of the windings being 32·8 cm. The coils were firmly clamped, after symmetry tests, with the planes of the windings horizontal at a mean distance of separation of 8·1 cm., sufficient to permit the passage of the rotor shaft, and their series connection could readily be switched from conjunction to opposition.

In order that the maximum mutual inductance between the rotor and the field coils should be capable of continuous variation by ± 1 %, five additional turns were wound round the outer central portions of the coils; each twin pair of turns, one on each field coil, contributed some 37 μH., the leads being brought, as shown at J in figure 10, to separate dial terminals, 0–5; furthermore, two turns (Cn) of constantan wire (S.W.G. 18) were also wound round each field coil and joined in series with a 1-ohm dial resistance-box in tenths so as to be in parallel with suitable rheostats, Rh_2, Rh_3, and thus provide, with fine adjustment, a continuous range of from 0 to 40 μH. This system, which is essentially that

described by Astbury (1938) in the construction of a standard mutual inductance, was most satisfactory, the impurity effect being negligible.

The complete rotor of the type described in § 2 is seen in figure 8. Each of the coils C and D was wound on a former having a channel of width 1·1 cm. and of internal diameter 11·1 cm., with 308 and 310 turns respectively of d.s.c. copper wire of S.W.G. 26; they were connected by twin bell flex which passed through the oak bed and hollow brass spindle to the insulated terminals a, b, c, d (figure 9) on the commutator M. After adjustment and fixing of the field coils for symmetry with respect to the horizontal axis of rotation, each rotor coil was separately adjusted in normal displacement from this axis, with the aid of flat

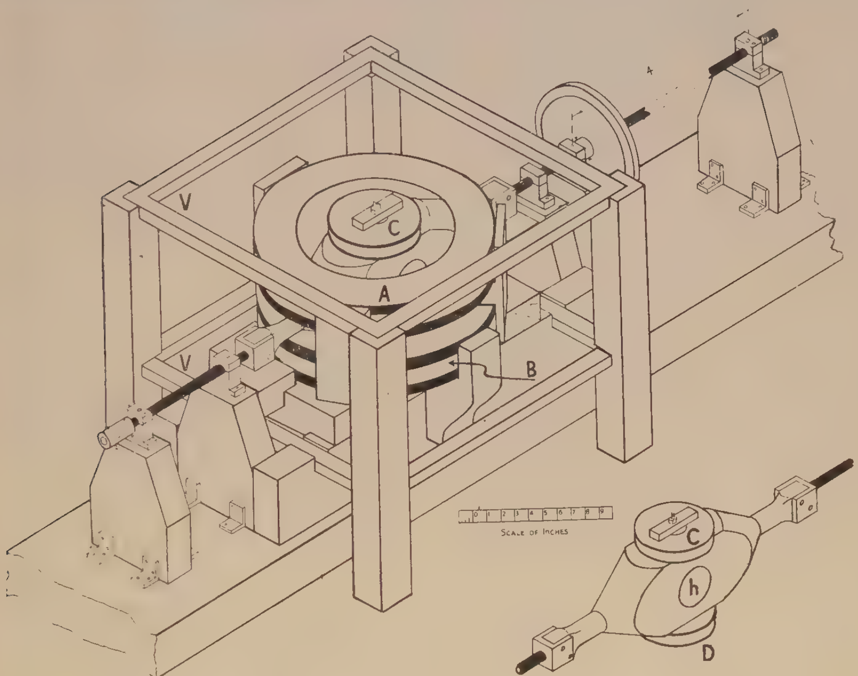

Figure 8. The field coils and rotor.

oak distance pieces, so as to obtain a symmetrical and maximum flat of mutual inductance with the plane of its windings horizontal and parallel to those of the field coils; the rotor coils were then connected at the commutator in series and conjunction and had then a maximum inductance with the field coils of 20,032·7 μH. (nominal), which could be varied with the aid of additional windings to any value between 19,806 μH. and 20,258 μH. Although the rotor was of larger size than that described in § 2, the "flat" of mutual inductance over a range well exceeding the largest possible short circuit was so highly satisfactory that it was decided, in view of the imminence of war, to proceed at once with absolute measurement; our intention of later determining the precise wave form of the

generator was in the end frustrated. The resistance of the rotor across the break terminals was 17·5 ohms and its self-inductance 38,510 μH.

The commutator M and the shorting rotor N, with the brushes H, are seen in figure 9. The ebonite-filled gaps between the brass halves of the commutator are about 1° of arc. The final form of shorting rotor consisted of an ebonite cylinder 6 cm. long and of 5 cm. diameter, through the centre of which passed a stout brass tube fitted with set screws which served to secure the whole to the hollow shaft conveying the rotor leads to the commutator. The ebonite cylinder was covered with a tightly fitting brass tube of 5·6 cm. external diameter

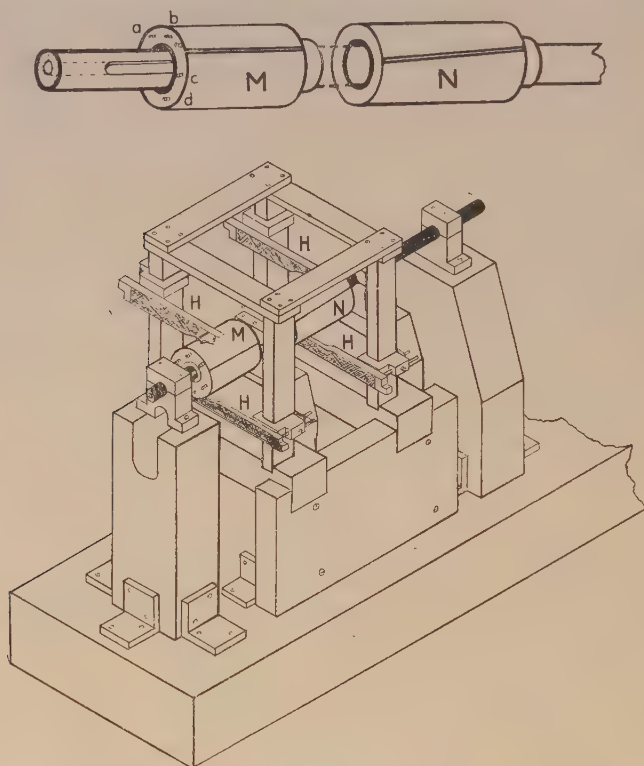

Figure 9. The commutator and shorting rotor.

which, between the binding screws near the edges, had two straight cuts 4 mm. wide, the one parallel to the axis of the cylinder and the other slightly inclined thereto, so as to divide the whole into two portions, the larger of which varied in angle over the length of the cylinder from 183° to 190°, and the smaller from 159° to 152°. After filling the gaps with ebonite and rounding true, the rotor yielded the desired range of short circuit of some 4° to 9° between diametrically opposite brushes, the variation being obtained by merely sliding the rotor along the shaft and re-clamping. The spring brass brushes, with their light steel springs and sponge-rubber dampers, had slight adjustment in all directions;

they were the outcome of many trials and were most satisfactory, being preferable for constancy in angle of short circuit, even contact, and clear break at commutation, to the fiddle-bow brushes used in the preliminary investigation (Balls, 1938).

The horizontal axis of rotation lay in the magnetic meridian, and the earth's vertical flux through the rotor was neutralised by the large horizontal coils V, V, of sides 49 cm. by 55 cm., placed in the equivalent Helmholtz position and conveying a current of about 0·22 amperes.

The drive and speed regulation was similar to that already used by us and described elsewhere (Nettleton and Balls, 1935; Balls, 1938.) In this case, however, a special valve-maintained tuning fork with very fine adjustment of frequency around 375 was used with valve amplification to supply current to the coils of a 30-toothed-wheel synchronous motor as well as to a unilateral neon lamp which illuminated a 30-line stroboscopic disc on the shaft. At first, the fork frequency was determined by taking the revolutions with two twin telephone counters over prolonged periods of synchronization of the motor, detached from the main shaft in a manner already described (Balls, 1938.) Later, the fork was accurately adjusted to a frequency of 375 at 18°·7 C. by use of the 1000-cycle note broadcast by the N.P.L., which was amplified to light a second neon lamp and so obtain, through a low-power telescope, a stroboscopic view of an 80-line disc on the shaft. Unfortunately, this note was only broadcast monthly and, owing to interference with other lines, it was not possible to have the 1000-cycle note of the P.O. Research Station transmitted to us over the telephone. For intermediate checks we had to make use of the daily long-wave transmission of Deutschlandsender which, owing to local interference, gave a pattern less clear and more difficult to follow than that given by the N.P.L. signal. During the course of two months, during which all the more important tests were made, the frequency of the fork remained so constant that the only change necessary to maintain the standard speed of revolution was a change in temperature of the fork from 18°·7 C. to 18°·9 C.

The chief features of the electrical circuit are shown in figure 10. The standard mutual inductance was $10{,}007{\cdot}5 \pm 0{\cdot}1$ c.g.s. μH. at 10 cycles and was equivalent under D.C. to a reading of 10,011·2 nominal μH. on the Campbell inductometer. Thus the {rotor, field-coil} mutual inductance when balanced against the standard together with the inductometer at a stud reading of 10,000 and m divisions on the scale is $20{,}003{\cdot}8 + m$ true μH. The circuit permits of tests for interaction between the inductometer and the standard, which was carefully eliminated.

A useful expedient is the key P_{11} in one of the brush connections of the shorting rotor, with leads, V_1, V_2, therefrom to wander plugs and the lead V_0 from r_2 serving for connection with an ohmmeter. This permits of individual tests of the contacts of the commutator and shorting rotor and enables the latter to be set symmetrically to cover the breaks in the former when the planes of all coils are parallel. Moreover, in measuring the sweeps of inductance between the rotor and field coils, the leads, V_0, V_1, enable the former to be set sharply in turn at

the four boundaries of the short circuits, which are readily put out of action while the inductometer readings are taken.

The central auxiliary circuit, including R, S and W, serves, with slight changes, for the estimation of θ, the mean angle of short circuit, for the potentiometer test for the compensation of the self-inductance of the rotor and for preliminary observation of the main balance before switching over to the more delicate detectors. The angle θ, though not required directly, is of interest and enables the resistance r to be adjusted by virtue of equation (26) and by means of the box r_2 prior to an experimental compensation test; it is found by switching on the accumulator E_2 through the appropriate keys to the shorting rotor with r_2 infinite, and obtaining a full-scale static reading of 120 divisions on the microammeter A_2 by adjustment of S and W. The rotor is turned to break contact

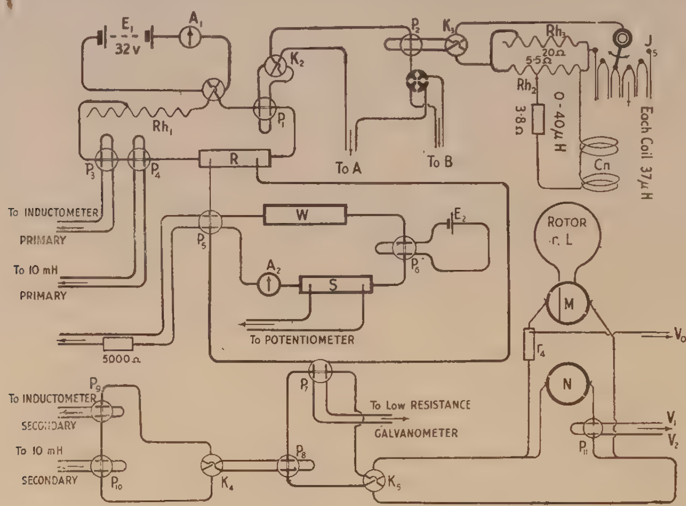

Figure 10. The experimental circuit.

through N, the voltage at E is increased m times without sensible change of resistance, and the deflection d is observed on the ammeter on spinning the rotor at any speed sufficient for steady reading. We have then $\theta = 3d/2m$. Potentiometer determinations of θ have also been made but are less simple.

In performing the compensation test, the effect of the earth's magnetic field is neutralized by adjustment of the current through the coils V, V (figure 8), as judged by the absence of deflection of a low-resistance galvanometer connected via the keys P_7, P_8 (figure 10) to the spinning rotor. The rotor is then brought to rest in any position in which the shorting rotor is inoperative, and a current is sent through it from the accumulator E_2 via W, R and S. With S = 900 ohms and W approximately the same, the e.m.f. across S of about a volt is balanced on the Weston-cell terminals of a thermoelectric potentiometer by adjustment of W and of the standard cell slide-wire. The motor is then set spinning at the standard speed and the potentiometer balance tested. If the dynamical balance reveals a loss of average current by fall in the e.m.f. across S, the effect of self-

inductance prevails and r must be increased at r_2. When r is correct, the potentiometer reveals a clear gain at slow speeds which diminishes as the rotor gathers pace and which vanishes on synchronization. the dynamical balance being identical with the statical for either direction of the reversing key K_5. The value of r, easily sensitive to 2 %, was then raised by 0·2 ohms to 0·5 ohms in accordance with equation (23).

In the main experiments, the rheostats Rh_1 are adjusted for a current of from 0·6 to 0·8 amperes to flow from the accumulators E_1 of 32 volts through the resistor R and the field coils, with the inductometer and standard primaries out of circuit. On spinning the rotor at standard speed, with the earth's field neutralized, approximate balance is obtained in the micro-ammeter A_2 before rocking over the key P_5 through some 5000 ohms on to, first, a pilot galvanometer and finally a high sensitivity galvanometer of period 21 sec. and resistance 3000 ohms. The coils of all the detectors were set symmetrically so that no deflection was produced by alternating current; the broadening of the line of light was inappreciable in the final balance with the sensitive galvanometer. The criterion of balance, attained by varying the mutual inductance with the coil switch J, if necessary, and the rheostats Rh_2, Rh_3, is no permanent movement of the spot of light on reversing the key K_1, a procedure which keeps constant any residual effect due to the earth's field and other causes. No detectable change in the balance was ever observed if taken in this way with both the keys K_2 and K_5 reversed. The balance is confirmed by repeated reversals on the reverser K_1 at intervals of a minute or more, and any irregularity is usually overcome at once by applying a light brush moistened with petrol to the commutator or shorting rotor. Using a current of 0·8 amperes, and with the resistance of the detector circuit at 8000 ohms, the sensitivity was such that a full-scale deflexion of 75 cm. was obtained in one minute on the large scale common to both galvanometers at each reversal of the key K_1 when the mutual inductance was changed from the exact balancing value by 5·0 μH. The stability of the balance is closely bound up with the improvements made since the preliminary investigation in the accuracy of spinning between close bearings of the commutator and the shorting rotor, in the design of the latter and of the brushes, and in the use of non-abrasive cleaners and polishers.

Immediately after the balance, the rotor is brought to rest and, with the aid of key P_{11} and an ohmmeter set sharply on the four boundaries of short circuit, the inductometer readings are taken to 0·1 μH., using the same current as in the main experiment. For this test, the inductometer and standard primaries are switched into the main circuits at P_3 and P_4; P_5 and P_{11} are open and the rotor is joined through the two secondaries to a sensitive low-resistance galvanometer of 3 sec. period. By taking the mean of these four readings in estimating M_0, not only have we a true measure of the actual flux operative, but error due to a fixed mutual inductance between the field coils and the portion of the secondary circuit between the commutator brushes and the reversing key K_5 is eliminated.

§ 5. EXPERIMENTAL RESULTS

The data of a typical experiment are given in table 1 below:—

Table 1. Data of experiment 15

Speed of revolution at 18°·9 C.	$12{\cdot}5000_0$ rev./sec.
Angle of short circuit θ	$7°{\cdot}4 \pm 0°{\cdot}1$
Compensation resistance for shorting rotor r	$24{\cdot}0 \pm 0{\cdot}3$ ohms
Primary current C_0	0·8 amperes
Detector circuit resistance F	8000 ohms
Temperature of fork	19°·0 C.
Temperature of ohm	19°·6 C.
Mean scale inductometer reading m	$7{\cdot}3_0$ nom. μH.
Measured mutual inductance	$20011{\cdot}1_0$ true μH.
Correction for fork temperature	$-0{\cdot}2_2$ μH.
M_0	$20010{\cdot}8_8$ μH.
$R = 4M_0 \,.\, n \times 10^9$	$1{\cdot}00054_4$ ohm
R in international ohms at 19°·6 C.	$1{\cdot}00002_8$ ohm
Value of one international ohm	$1{\cdot}00051_6$ ohm

Table 2. Results of consecutive experiments

Exp.	$\theta°$ of arc.	r ohms	m nom. μH.	$dT°$ C.	Temp. of ohm in ° C.	Value of ohm in c.g.s. units $\times 10^9$	Value of ohm in internat. units	Difference
1	5·2	33·5	$6{\cdot}7_0$	0·0	18·8	$1{\cdot}00052_5$	$1{\cdot}00000_4$	$\cdot00052_1$
2	5·2	33·5	$6{\cdot}6_0$	0·0	19·0	$1{\cdot}00052_0$	$1{\cdot}00001_0$	$\cdot00051_0$
3	5·2	33·5	$7{\cdot}0_5$	0·2	19·1	$1{\cdot}00052_1$	$1{\cdot}00001_3$	$\cdot00050_8$
4	5·2	33·5	$6{\cdot}6_2$	0·1	18·9	$1{\cdot}00051_0$	$1{\cdot}00000_7$	$\cdot00050_3$
5	6·5	27·3	$7{\cdot}8_0$	0·5	19·4	$1{\cdot}00052_5$	$1{\cdot}00002_2$	$\cdot00050_3$
6	6·5	27·3	$7{\cdot}9_8$	0·4	19·3	$1{\cdot}00054_5$	$1{\cdot}00001_9$	$\cdot00052_6$
7	6·5	27·3	$7{\cdot}8_5$	0·4	19·3	$1{\cdot}00053_9$	$1{\cdot}00001_9$	$\cdot00052_0$
8	8·65	20·2	$7{\cdot}9_3$	0·5	20·0	$1{\cdot}00053_1$	$1{\cdot}00004_0$	$\cdot00049_1$
9	8·65	20·2	$4{\cdot}4_0$	−0·75	18·1	$1{\cdot}00049_3$	$0{\cdot}99998_3$	$\cdot00051_0$
10	8·4	20·7	$6{\cdot}2_5$	0·0	18·9	$1{\cdot}00050_3$	$1{\cdot}00000_7$	$\cdot00049_6$
11	8·4	20·7	$6{\cdot}4_5$	0·0	18·7	$1{\cdot}00051_3$	$1{\cdot}00000_1$	$\cdot00051_2$
12	7·4	24·0	$6{\cdot}6_2$	0·0	19·4	$1{\cdot}00052_1$	$1{\cdot}00002_2$	$\cdot00049_9$
13	7·4	24·0	$6{\cdot}4_2$	0·0	18·9	$1{\cdot}00051_1$	$1{\cdot}00000_7$	$\cdot00050_4$
14	7·4	24·0	$6{\cdot}5_5$	0·1	19·1	$1{\cdot}00050_7$	$1{\cdot}00001_3$	$\cdot00049_4$
15	7·4	24·0	$7{\cdot}3_0$	0·1	19·6	$1{\cdot}00054_4$	$1{\cdot}00002_8$	$\cdot00051_6$
16	7·4	24·0	$7{\cdot}7_8$	0·3	19·8	$1{\cdot}00054_6$	$1{\cdot}00003_4$	$\cdot00051_2$
						Mean difference . .		$\cdot00050_8$
		One international ohm = 1·00051 ohm.						
17	7·4	24·0	$2{\cdot}3_0$	0·0	19·0	$1{\cdot}00030_5$	$0{\cdot}99981_0$	$\cdot00049_5$
18	7·4	24·0	$-3{\cdot}4_8$	0·0	19·0	$1{\cdot}00001_6$	$0{\cdot}99951_0$	$\cdot00050_6$
19	5·2	33·5	$69{\cdot}1_0$	0·2	20·0	$1{\cdot}00362_3$		

In table 2 are first given the results of sixteen consective direct experiments spread over five weeks. Experiments 17 and 18 were performed with 5000 ohms and 2000 ohms, respectively, in parallel with the international ohm. Experiment 19 was performed between experiments 4 and 5 with an old laboratory german-silver Board of Trade ohm. m is the mean reading on the scale of the inductometer of the four boundary settings; dT is the temperature of the fork above that at which the speed of revolution was last confirmed at 12·5 rev./sec.

The consistency of the method, allowing for temperature changes, is shown by the extreme variation of 3·5 parts in 10^5 in 16 experiments. The tenth-ohm box linking the constantan coils Cn (figure 10) to the rheostats Rh_2, Rh_3 was often varied to prevent any fore-knowledge of the rheostat settings at which balance might be expected.

§ 6. CONCLUDING REMARKS

This paper is the last of the series of investigations on absolute measurement of electrical resistance carried out in recent years at Birkbeck College. This method in its present form far exceeds in accuracy any of our previous methods, in particular that of the rotating coil recently improved by one of us (Balls, 1938). This is because of the great sensitivity, not yet pushed to its limit, the adoption of the method to measure directly a resistance of an international ohm, and the need for accurate measurement only of a frequency of 1000 cycles and of a mutual inductance of very closely 20,000 μH. on a remarkable "flat." The 2% experimental compensation test is an important feature of the method, and the agreement here with theory is satisfactory in showing the insignificance of additional effects such as would arise from imperfect contact at the brushes. There would be little difficulty in attaining here an accuracy approaching 0·5 %, the chief advantage of which would be manifest in the stability of the main balance.

We regret that we are unable to reconstruct the apparatus and carry out further improvements—for we believe the method is capable of high precision. As it is, we had the satisfaction of knowing that the most accurate method of calibrating an ohm in our laboratories was by this rapid method of measuring its resistance directly in c.g.s. units.

§ 7. ACKNOWLEDGEMENTS

We express our gratitude to Prof. J. D. Bernal, M.A., F.R.S., for the help and encouragement that he has given us throughout this investigation. We also wish to thank Mr. W. Wilson, B.Sc., of the National Physical Laboratory, Dr. W. Ehrenberg, of Electrical and Musical Industries, Ltd., Hayes, and Mr. S. Baker, Mr. H. G. Bell and Mr. R. A. Dobb, of Birkbeck College, for the help and advice that they have given us.

REFERENCES

ASTBURY, N. F., 1938. *Phil. Mag.* **25**, 290.
BALLS, E. G., 1938. "The absolute measurement of electrical resistance by new rotating coil methods." Thesis (London).
CURTIS, H. L., 1937. *Elect. Measurements* (London: McGraw-Hill), p. 177.
LLEWELLYN, F. H., 1934. *Proc. Phys. Soc.* **46**, 839.
NETTLETON, H. R. and BALLS, E. G., 1935. *Proc. Phys. Soc.* **47**, 58.
ROSA, E. B., 1909. *Bull. Bur. Standards*, **5**, 499. See also GLAZEBROOK, *Dict. of Applied Physics*, vol. 2, p. 226.

NOTE ON ELECTRIC AND MAGNETIC DIMENSIONS

BY G. D. YARNOLD, M.A., D.PHIL.,
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ABSTRACT. Attention is drawn to the fact that the dielectric constant K and the magnetic permeability μ of a medium are essentially dimensionless. From the definitions given for these quantities the usual forms of the equations expressing the inverse-square laws of force are readily derived. The dimensions of the constant factor unity in each of these equations must not, however, be associated with K and μ.

THREE extremely interesting papers which have recently appeared in these *Proceedings* have raised again the vexed question of the physical dimensions of electric and magnetic quantities, and in particular of the dielectric constant K and the magnetic permeability μ of a medium. Thus, Brown (1940, 1941) maintains that both K and μ, being defined as ratios of quantities of similar dimensions, are themselves necessarily dimensionless; while Duncanson (1941) regards K and μ as possessing dimensions. The whole difficulty appears to arise from the confusion of two quite distinct quantities, viz., the constant which must always be included in the equation expressing an inverse-square law of force and the quantity known as the dielectric constant or the permeability of the medium, as the case may be. Of these, the former is found to possess dimensions, while the latter is by definition dimensionless. Benham, in the discussion on Brown's earlier paper (1940), noted this same point, but his remarks do not appear to have received the attention they deserve.

Adopting the usual nomenclature, the two inverse-square laws are expressed as follows:

$$\text{force between point charges } \textit{in vacuo} \quad \propto \frac{q_1 q_2}{r^2},$$

$$\text{force between point poles } \textit{in vacuo} \quad \propto \frac{m_1 m_2}{r^2}.$$

It should be noted that these laws have been established by experiment only in the case of charges or poles in air or vacuum. No person has any direct experimental knowledge of the forces between charges or poles " embedded " in any material medium other than a gas, and the extension of the laws to the general case rests entirely on other considerations.

The laws of force may be written as equations by introducing two constants a and b. Thus, referring still to vacuum,

$$F=a\frac{q_1q_2}{r_2} \quad \text{and} \quad F=b\frac{m_1m_2}{r^2}.$$

All that can be said at present about the dimensions of a, b, q and m is that aq^2 and bm^2 each have the dimensions of a force multiplied by the square of a distance, i.e. $[\mathrm{ML^3T^{-2}}]$ in terms of the usual three indefinables. It may be shown, however, that the product ab has the dimensions of the square of a velocity, i.e. $[\mathrm{L^2T^{-2}}]$. Thus, while there is still no information concerning the dimensions of a and b separately, it is clearly unjustifiable to treat them as dimensionless.

The introduction of a medium other than vacuum is most conveniently made in the case of the electrical problem by considering the change in the capacity of a condenser brought about by completely filling the space between the plates with the medium. The definition of the dielectric constant K of the medium is then

$$K=\frac{\text{capacity of condenser with dielectric}}{\text{capacity of condenser without dielectric}}.$$

It follows that the difference of potential between the plates of a condenser carrying fixed charges is reduced by the medium in the ratio $1/K$, and that the law of force for point charges embedded in the medium is

$$F=\frac{a}{K}\frac{q_1q_2}{r^2}.$$

in which K is, by definition, a dimensionless number. It is assumed implicitly that the introduction of a small charge produces no disturbance of the electrical conditions in the medium ; in other words, the medium is treated from a strictly classical point of view.

In the magnetic problem, the medium is introduced by considering the mechanical forces acting on a small hypothetical test pole when placed in turn in an evacuated long, thin, cylindrical cavity and in an evacuated short, flat, cylindrical cavity in the medium, the axis of the long cavity being parallel to the direction of the magnetizing field, and the axis of the short cavity parallel to the direction of magnetization of the medium. The definition of the magnetic permeability μ of an isotropic medium is then

$$\mu=\frac{\text{mechanical force on test pole in short flat cavity}}{\text{mechanical force on test pole in long thin cavity}}.$$

Thus μ also is defined as a dimensionless number, and the law of force for point poles embedded in the medium may be obtained as follows:—

Consider an isotropic medium of permeability μ, containing two short flat cavities bounded by four parallel infinite planes A, B, C, D (figure 1). Let the flat cavities be connected by a long thin cavity parallel to the direction of the magnetizing field and perpendicular to the set of planes, and suppose all the cavities to be evacuated. A small positive magnetic pole may be transported slowly from a point Z between planes C and D to a point X between planes A and B, either via the long thin cavity or by the direct route through the medium. The work done by the operator is the same in either case. Hence the mechanical

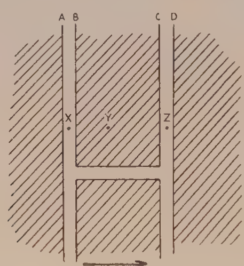

Figure 1.

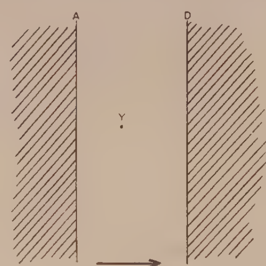

Figure 2.

force acting on the pole when placed at any point Y in the medium is the same as that acting on the pole when placed in the long thin cavity. Now let that part of the medium lying between planes B and C be removed (figure 2). Since the planes A and D are infinite while their separation is finite, the mechanical force on the pole when placed at Y is the same as that which previously acted on it when placed at X or Z. Thus, by the definition of the magnetic permeability, the force acting on the small pole when embedded in the medium is equal to $1/\mu$ of the force acting on it in vacuum, provided the strength of the magnetizing field is unchanged. The general form of the law of force between point poles is thus

$$F = \frac{b}{\mu} \frac{m_1 m_2}{r^2}.$$

It is unfortunate, in view of the formal similarity between the laws of force for electric charges and magnetic poles, that exactly analogous definitions cannot be adopted for K and μ. The reason for this no doubt lies in the fact that while on the atomic scale there is such a thing as an electric charge, there is no such thing as a magnetic pole. All modern work on magnetic phenomena shows that magnetic moment rather than magnetic pole is the fundamental entity, and that this in turn can be described in terms of moving electric charge.

The customary definitions of the electrostatic unit of charge and the unit of magnetic pole strength are designed to make the constants a and b each equal to unity when the forces are measured in dynes and the separation of the charges

or poles in centimetres. The laws of force between point charges and point poles placed in a medium then reduce respectively to their usual forms:

$$F=\frac{q_1q_2}{Kr^2} \quad \text{and} \quad F=\frac{m_1m_2}{\mu r^2},$$

but it must always be borne in mind that these equations, as they stand, are dimensionally incorrect, since a constant equal to unity but possessing unknown dimensions is omitted from each. No corresponding ambiguity exists in the statement of the gravitational law of force, since in this case the constant G is retained and is recognized as possessing dimensions.

The confusion in electric and magnetic units arises because the dimensions proper to a and b respectively in the above equations are commonly associated with K and μ. In particular, it is thoroughly misleading to state that the square of the velocity of light is equal to the reciprocal of the product of the dielectric constant and the magnetic permeability of free space. In fact, both the dielectric constant and the permeability of free space are, by definition, dimensionless and equal to unity, and it is the product ab which may be identified with the square of the velocity of light, provided, of course, that the same system of electrical units is used throughout. It is felt that the general adoption of a notation similar to that employed above would reduce the present confusion.

From what has been said, it is clear that electric and magnetic dimensions cannot be expressed in terms of the three fundamental quantities mass, length and time only, and there appears to be much to commend the acceptance of electric charge as an additional indefinable. The dimensions of the constant a are, then, $[ML^3T^{-2}Q^{-2}]$ and those of b are $[M^{-1}L^{-1}Q^2]$, while K and μ are dimensionless. The dimensions of other quantities remain as given by Duncanson, with the exception that electric and magnetic induction have respectively the same dimensions as the corresponding field strengths.

The fact that precise dimensions have been assigned to the constants a and b does not in any way imply that the phenomena of electric and magnetic repulsion or the properties of free space are completely understood. The adoption of the four indefinables M, L, T and Q, each of which appears incapable of *adequate* representation in terms of the other three, is merely a matter of convenience, and makes possible a consistent system of electric and magnetic dimensions.

REFERENCES

BROWN, 1940. *Proc. Phys. Soc.* **52**, 577.
BROWN, 1941. *Proc. Phys. Soc.* **53**, 418.
DUNCANSON, 1941. *Proc. Phys. Soc.* **53**, 432.

DISCUSSION

Mr. S. GILL.* Dimensions of common physical quantities are usually written in terms of the fundamental quantities length, time, and mass, together with

* It seems worthy of record that Mr. Gill is only 15 years of age. (ED.)

either permeability or permittivity. These systems are awkward owing to the two alternative ways of representing electrical and magnetic quantities, and the fact that fractional indices often occur.

The choice of fundamental quantities is arbitrary, and I find that by taking as fundamentals length, time, electric charge and magnetic pole-strength, not only are fractional indices avoided, but a striking similarity between the dimensions of corresponding electrical and magnetic quantities is found. If any pair of corresponding magnetic and electrical quantities is taken, e. g. intensities of fields, the indices of length and time are found to be identical, while those of charge and pole-strength are interchanged. Moreover, in the case of " pure " physical quantities, the indices of charge and pole-strength are the same, so that interchanging has no effect.

	Length	Time	Charge	Pole-strength
Mass	−2	1	1	1
Momentum	−1	0	1	1
Force	−1	−1	1	1
Energy	0	−1	1	1
Moment of inertia	0	1	1	1
Angular momentum	0	0	1	1
Permeability	−1	1	−1	1
Field-strength (magnetic)	−1	−1	1	0
Magnetic induction	−2	0	0	1
Rate of change of flux	0	−1	0	1
Magnetic potential	0	−1	1	0
Permittivity	−1	1	1	−1
Electric field-strength	−1	−1	0	1
Electrical induction	−2	0	1	0
Electrical current	0	−1	1	0
Electrical P.D.	0	−1	0	1

This shows clearly the fundamentally complementary nature of electrical and magnetic phenomena, although in practice this is masked by the fact that they are manifested in such widely differing ways.

DIAMAGNETIC SUSCEPTIBILITY OF METHANE

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ABSTRACT. Theoretical calculations of the diamagnetic susceptibility of methane CH_4 have been made on the basis of the molecular-orbital and electron-pair approximate wave functions. The results are compared with the self-consistent-field value, and it is shown that in this latter an error of about 25 % is due to the preliminary averaging of the positive charges on the four protons. The new values agree well among themselves, but are still considerably larger than the experimental value. The reasons for this are briefly discussed.

VERY few theoretical calculations of the diamagnetic susceptibility of molecules have been reported in the literature. This is directly due to our ignorance of reliable wave functions. Indeed, the diamagnetic susceptibility depends immediately upon the wave function, and the various types of approximation employed in calculating this latter may be tested by a comparison of the predicted susceptibilities.

Interest in this problem has recently been revived by the stimulating work of Buckingham, Massey and Tibbs (1941). These authors have dealt very fully with the methane molecule, using the self-consistent-field method (s.c.f. method) of Hartree. But in order to employ the customary technique of the s.c.f. approximation, it was necessary to average the four protons round the surface of a sphere with the carbon atom as centre; in this manner they procured the requisite spherical symmetry. There is probably no great error involved in this averaging process, especially with molecules so nearly spherical as CH_4, even though, as these authors explain, on account of the relatively slow motion of the nuclei compared with that of the electrons, it is a more drastic approximation than that used in the normal s.c.f. method for atoms. It is, however, desirable that we should test this averaging process as fully as we can, especially in view of its possible application to other molecular systems, and in this note, therefore, we report alternative calculations of the diamagnetic susceptibility, using molecular wave functions in which no averaging of the protons has taken place.

The formula for the diamagnetic susceptibility is (Van Vleck, 1932)

$$\chi_{\text{mol}} = -\frac{\boldsymbol{N}e^2}{6\boldsymbol{m}c^2}\Sigma\overline{r^2} + \tfrac{2}{3}\boldsymbol{N}\sum_{n'\neq n}\frac{|\,m^\circ(n',n)\,|^2}{\boldsymbol{h}\nu_{n'n}},$$

in which $\boldsymbol{N}$ is Avogadro's number and the other symbols have their usual meanings.

The summation in the first term is over all the electrons in the molecule, and in the second term over all non-diagonal matrix components of the electric moment. In the case of spherical symmetry this latter term is identically zero, and it is very unlikely to be large for methane, if we measure r from the carbon nucleus. We shall accordingly neglect this contribution and thus follow the course adopted by Buckingham, Massey and Tibbs. Our problem is therefore reduced to a calculation of the mean value of r^2 for each electron present.

Our first molecular wave functions are the molecular-orbital wave functions (m.o. approximation) determined by the present writer some years ago (Coulson, 1937). According to these calculations, the ten electrons are divided into three groups. The first group contains what are essentially the two K electrons of carbon, whose normalized wave functions are

$$\psi(C:1s)=\sqrt{(\gamma^3/\pi)}e^{-\gamma r}, \qquad \gamma=5{\cdot}7.$$

We are using atomic units throughout. The second group, called by Mulliken $[s]$, has the complete symmetry of the molecule and contains two electrons with normalized wave functions:

$$\psi[s]=N_s\{8{\cdot}22\,\psi(C:2s)+\psi(H_a)+\psi(H_b)+\psi(H_c)+\psi(H_d)\},$$

where

$$\psi(C:2s)=\sqrt{(c_s{}^5/3\pi)}re^{-c_s r}, \qquad c_s=2{\cdot}98,$$

$$\psi(H_a)=\sqrt{(1/\pi)}\,e^{-r_a},$$

$$N_s=0{\cdot}10145.$$

The third group, called by Mulliken $[t]$, holds two electrons in each of three orbitals, a typical member being

$$\psi[t_x]=N_t\{1{\cdot}70\,\psi(C:2p_x)+\psi(H_a)+\psi(H_b)-\psi(H_c)-\psi(H_d)\},$$

where

$$\psi(C:2p_x)=\sqrt{(c_p{}^5/\pi)}xe^{-c_p r}, \quad c_p=1{\cdot}62,$$

$$N_t=0{\cdot}3259,$$

and the x axis bisects the line joining H_a and H_b. The other two members of this series are obtained by an obvious cyclic interchange.

It is not difficult to evaluate $\overline{r^2}$ for each group of wave functions. The integrals involved, some of which are fairly complex, may all be obtained from tables of such integrals prepared by the writer (Coulson, 1941). The mean values, in atomic units, are given below:

Wave function	$\psi(C:1s)$	$\psi[s]$	$\psi[t]$
$\overline{r^2}$	0·092	1·527	5·018

It will be noticed that the $[s]$ type orbit is much more concentrated than the $[t]$ type, and, in fact, almost all the susceptibility arises from the six $[t]$ electrons. The value of χ deduced from these mean values is

$$\chi_{\text{mol}}=-26{\cdot}6\times10^{-6}\ \text{e.m.u.}$$

Before we discuss this result we must deal with the second approximation. This is the electron-pair method (e.p. method) of Heitler and London. The electrons are taken in pairs and exchange is allowed for between the two members of a pair, but not between members of different pairs. There are two inner K-shell electrons similar to those described above in the m.o. approximation, and then four pairs (one pair for each C-H bond). Each of these pairs represents a homopolar bond and has a wave function such as

$$\Psi(\sigma)=N_\sigma\{\psi(\sigma:1)\psi(H_a:2)+\psi(\sigma:2)\psi(H_a:1)\},$$

where N_σ is a normalizing constant;

and $\psi(H_a)=\sqrt{(\beta^3/\pi)}\ e^{-\beta r_a}$,

$$\psi(\sigma)=\tfrac{1}{2}\psi(C:2s)+\frac{\sqrt{3}}{2}\psi(C:2p).$$

$\psi(\sigma)$ is, of course, the Pauling tetrahedral orbit, and

$$\psi(C:2s)=\sqrt{(\alpha^5\beta^5/3\pi)}re^{-\alpha\beta r},$$
$$\psi(C:2p)=\sqrt{(\alpha^5\beta^5/\pi)}xe^{-\alpha\beta r}.$$

The value of α is taken to be that given by Slater (1932), viz., $\alpha=1\cdot625$. We have, however, not retained the Slater exponent α unaltered, but have included, both in $\psi(\sigma)$ and $\psi(H)$, an extra factor β. This is introduced because, as Coulson and Duncanson (1941) have shown, some such factor is necessary to allow for different screening in passing from atomic to molecular wave functions. A value $\beta=1$ corresponds to the normal Slater functions for atoms, but a value $\beta=1\cdot1$ is probably more accurate for molecules. For the present, then, we retain $\beta=1\cdot1$, and later we shall discuss the effect of varying β between $1\cdot0$ and $1\cdot2$. We do not require to introduce this factor for the 1s electrons, which are substantially unaffected by molecular formation.

Using these wave functions, we find that, in atomic units,

$$\overline{r^2(1s)}=0\cdot092, \qquad \overline{r^2(\sigma)}=4\cdot313.$$

The resulting value for χ_{mol} is $\chi_{\text{mol}}=-27\cdot7\times10^{-6}$ e.m.u. The various values of χ_{mol} are collected in the table.

Table. Values of the diamagnetic susceptibility

Molecular-orbital	$-26\cdot6\times10^{-6}$ e.m.u.
Electron-pair ($\beta=1\cdot1$)	$-27\cdot7\times10^{-6}$ e.m.u.
Self-consistent-field	$-33\cdot2\times10^{-6}$ e.m.u.
Experimental value (Bitter, 1929)	$-12\cdot2\times10^{-6}$ e.m.u.

We may make several deductions from this table. First, the agreement between the m.o. and e.p. calculations is remarkably good. It is true, unfortunately, that the m.o. method allows for ionicity but not for exchange, whereas the e.p. method allows for exchange but not for ionicity, and in each case the neglected factor would reduce the numerical value.

The s.c.f. method of Buckingham, Massey and Tibbs may be compared with the m.o. method, which it resembles very closely, since they both allow for ionicity but not for exchange. It is seen that the s.c.f. value is about 25 % larger than the m.o. value. This may be interpreted as showing that the effect of averaging the four protons is to give too diffuse a charge cloud with too large a value of $\overline{r^2}$. We may expect a similar kind of result for other molecules of this type, and the averaging process will probably need some modification if it is to be applied to heavier molecules, such as CCl_4.

The values obtained in this paper with the m.o. and e.p. approximations, though less than that obtained from the s.c.f. approximation, are still about twice the experimental value; this factor, which is somewhat larger than that usually found with atoms, must be due chiefly to approximations in the wave functions. Indeed, there is every reason to believe that, as with atoms, if we could develop more accurate wave functions we could reduce the error very considerably. It is a pity that satisfactory molecular wave functions are so laborious to compute. In the present type of problem the presence of r^2 in all the integrands reveals that the largest influence comes from the outer regions of the molecule in which the detailed form of the wave functions is known with least reliability.

In conclusion it may be wondered whether the value of χ_{mol} found by the e.p. approximation is sensitive to the presence of the scale factor β, and, if so, whether there is much significance to be attached to the corresponding value in the table. We have therefore made equivalent calculations, taking $\beta = 1{\cdot}0$, $1{\cdot}1$ and $1{\cdot}2$, and the results are shown below:

$\beta =$	$1{\cdot}0$	$1{\cdot}1$	$1{\cdot}2$
$\chi_{\mathrm{mol}} =$	$-31{\cdot}0$	$-27{\cdot}7$	$-25{\cdot}3 \times 10^{-6}$ e.m.u.

The appropriate value of β is almost certain to lie within this range. Arguing by analogy from the case of molecular hydrogen H_2, we should expect β to lie between $1{\cdot}1$ and $1{\cdot}2$. This table shows that any plausible alteration in the value of β is inadequate to explain the difference between theory and experiment. This must be due, not so much to inaccuracy in the finer details of the wave functions, as to the coarser approximations, in which we neglect, on the one hand, ionic terms, or, on the other hand, the full determinantal symmetry of the molecular wave function.

REFERENCES

Bitter, 1929. *Phys. Rev.* **33**, 388.
Buckingham, Massey and Tibbs, 1941. *Proc. Roy. Soc.* A, **178**, 119.
Coulson, 1937. *Trans. Faraday Soc.* **33**, 388.
Coulson, 1941. *Proc. Camb. Phil. Soc.* (To appear shortly.)
Coulson and Duncanson, 1941. *Proc. Camb. Phil. Soc.* (To appear shortly.)
Van Vleck, 1932. *Electric and Magnetic Susceptibilities*, p. 275 (Oxford: Clarendon Press).

THERMOSTATS EMPLOYING EXTERNAL SURFACE CONTROL

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Received 28 *October* 1941; *Communicated by Prof T. H. Laby, F.R.S.*

ABSTRACT. An introduction is given by T. H. Laby, F.R.S., in which the principles involved in the production of constant temperature are discussed. These principles have been applied to the design of temperature-controlling systems using an external resistance thermometer to control the heat supply, which may be radiant heat or heated air. The hunting in the system in each case is less than 1 second. Control of the temperature of apparatus to better than 0·01° c. has been obtained by using these methods.

INTRODUCTION.

By T. H. Laby, F.R.S.

In scientific experiments and industrial operations it is often necessary to control the temperature of an instrument or of chemical or other plant. Principles which apply to such control are discussed below, and methods are described in which the instrument whose temperature is controlled is heated by radiation or warmed air, the heating supply being turned on and off by a resistance thermometer which is wound on the surface of the instrument. The time-lag of the thermostat is small, and the temperature fluctuations at the surface penetrate only to a small depth and the internal temperature is constant.

Mr. V. D. Hopper and the writer (Laby and Hopper, 1939), in measuring the electronic charge, devised a method to maintain at a constant temperature the air in which the fall of oil drops was under observation, and it proved most effective for that purpose. The resistance thermometer formed an arm of a direct-current bridge. The light spot of the galvanometer connected to the bridge operated a photoelectric cell, and the amplified current from that cell actuated a relay which turned on and off two lamps whose radiation heated the instrument.

Mr. Hopper at this stage was appointed to the Physics Laboratory of the University of Western Australia, where he has made tests of the radiation and warm-air thermostats. He has devised a bridge circuit and used an amplifier which employs A.C. current.

The principles which apply to the maintenance of an instrument at a constant temperature a few degrees above the maximum ambient temperature may be

briefly analysed. It is noted that the ambient temperature not only changes daily and seasonally, but that, superimposed on these comparatively long-period temperature variations, there are fluctuations of much shorter period. The systems considered lose heat by conduction, convection and radiation to the surrounding bodies. Newton's law of cooling no doubt applies to the losses of heat, as the excess temperatures which determine them are small.

Heat is supplied to the system whose temperature is to be kept constant, at a rate controlled by the thermometer. In an ideal thermostatic system its rate of loss at every instant is equal to its rate of gain of heat, that is, there is no time-lag in the control.

In this introduction the following may be mentioned :

(1) Time-lag in the control of the heat supply.
(2) Depth of penetration of temperature fluctuations.
(3) Thermal shielding.
(4) External heating and thermometry.

Time-lag. In any thermostat there is a lag in time between a change in the temperature of the thermostat and the change in the heat supply. A general theoretical study of this time-lag in control systems has been given by Callendar, Hartree and Porter (1936). These authors show that for stable control of temperature, (1) the supply of heat to the thermostat must return quickly to its normal value, and (2) there must be positive damping in the controlling system. This means that the thermometer which controls the heat supply must follow temperature changes rapidly and regulate the heat supply with a minimum delay.

A control with an inherently small time-lag is described later. It is a resistance thermometer of fine electrically insulated wire wound on the surface of the instrument to be controlled and connected to a Wheatstone bridge. The system, consisting of the thermometer, bridge and galvanometer, has a time-lag determined by the thermal capacity of the resistance thermometer and the period of the galvanometer, but these can readily be made small.

Depth of penetration of temperature fluctuations. Numerical values of the depth to which fluctuations of temperature penetrate into solids are given in the table on p. 59. It will be seen that fluctuations of temperature of short period penetrate thermal insulators only to a small depth. A temperature fluctuation of 1 sec. period has an inappreciable effect at a depth of 2 mm. below the surface of a sheet of cork. While the penetration is much larger for a good conductor, an enclosure constructed of alternate layers of good and bad conductors is an effective shield against temperature fluctuations.

Thermal shielding. In an enclosure of alternate layers of conductors and non-conductors, the non-conductor eliminates fluctuations of short period, and the layer of good conductor brings all points inside it to the same temperature.

Heating and thermometry at external surfaces. In some types of thermostat, the heater is external to the thermostat and in others an internal heater is used,

the thermometer which regulates the supply of heat being placed inside the thermostat. With the thermometer in this position, temperature changes, at least sufficient to actuate the thermometer, occur in the region from which it is the purpose of the thermostat to exclude any temperature change.

If a mercury thermometer is used it cannot readily be placed outside the thermostat, but it may be recalled that Griffith constructed a thermostat enclosed by its mercury steel thermometer. If a resistance thermometer is used, it can be readily wound on the external surface of the instrument whose temperature is to be controlled. If this surface is a good conductor it is only necessary to cover a part of that surface.

The thermostat can be heated by radiation or a stream of heated air. The time-lag can be made small, and the surface fluctuations in temperature are damped out before penetrating far into the thermostat.

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§ 1. CONTINUOUS AND DISCONTINOUS METHODS OF CONTROL

The temperature-control system employed consists essentially of a heater A (see figure 1), a means B of transmitting heat from the heater to the apparatus, a thermometer C which measures and controls the entry of heat into the shielding system D, and the object E whose temperature is controlled.

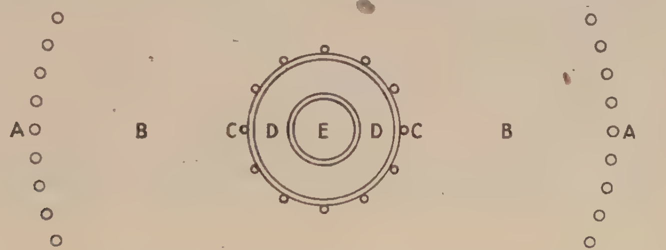

Figure 1. Temperature-controlling system.
A, heaters ; B, intervening medium ; C, thermometer ; D, shielding system ;
E, temperature-controlled section.

There are two methods, namely, the continuous and the discontinuous method, which can be employed using the above arrangement. With the continuous method, equilibrium is established between the amount of heat entering the apparatus and that leaving it. Let θ = temperature of the vessel under control; t = ambient temperature; $C(\theta - t)$ be the power emitted by the vessel (conduction, convection and radiation), where C is a positive constant; P = heating power provided by lamps and received by the vessel; $k = -dP/dT$ where k is a positive constant. For normal conditions

$$C(\theta - t) = P$$

and

$$\frac{dP}{d\theta} = C\left(1 - \frac{dt}{d\theta}\right),$$

$$\frac{d\theta}{dt} = \frac{C}{k+C}.$$

Thus the temperature fluctuations in the apparatus depend on those in the room and on the constants C and k. To reduce $d\theta/dt$ so that temperature control within a narrow range of temperature is obtained, it is necessary to make k large compared with C, but under all conditions there will be a change in apparatus temperature with change of ambient temperature.

Using the discontinuous method, the temperature at the surface of the vessel is allowed to fluctuate rapidly from $\theta+\Delta\theta$ to $\theta-\Delta\theta$ and the value of $\Delta\theta$ may be made practically constant by reducing the time-lag between heater and control. With the lag small, the temperature variations at C will be similar to those shown in figure 2 *b*, but if the lag is large, ***hunting*** is said to occur and the temperature variations increase as in figure 2 *c*. In the methods discussed later, the tempera-

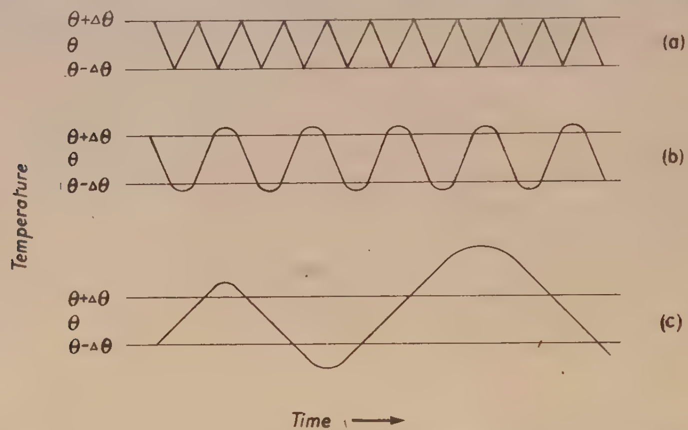

Figure 2. Graphs showing temperature measured by thermometer C for (*a*) no time-lag ; (*b*) small time-lag ; (*c*) large time-lag in control.

ture fluctuations are made to approximate to figure 2 *b*, and the value of $\Delta\theta$ is of the order of 0·002° c. Provided that there is some way of removing these fluctuations, the discontinuous method will be more suitable than the continuous one. The variations of temperature at C are conducted into the shield D. If we first consider this to be a semi-infinite solid whose surface temperature is given by $\theta=A\cos(\omega t-\epsilon)$ and whose initial temperature is $\theta=f(x)$, it can be shown (Carslaw, 1922) that if sufficient time has passed to allow the distribution of temperature to become purely periodic, and the influence of the initial distribution has passed away, the temperature of the material is given by

$$\theta=Ae^{-\sqrt{(\omega/2k)}}\cos(\omega t-\sqrt{(\omega/2k)}\,x-\epsilon),$$

where k is the thermal diffusivity of the substance $=K/c\rho$, K being its conductivity, c its specific heat and ρ its density. This shows that a temperature wave is propagated with unaltered period inwards, the amplitude of the wave of shorter period diminishing more rapidly than that of waves of greater period and also having a more rapid alteration in phase. The velocity of propagation is inversely proportional to the square root of the periodic time of the oscillation.

The table gives, for copper and cork, the maximum amplitude and the temperature variation at different distances from the surface when the period of variation of temperature is 1 second and 60 seconds.

Table. θ_x = maximum temperature change x cm. from surface

Substance	k	Period (sec.)	θ_x when x is 1 mm.	.5 mm.	1 cm.
Copper	1·13	1	·86	·44	·19
		60	·98	·89	·81
Cork	0·0017	1	·014	10^{-10}	10^{-15}
		60	·57	·06	4×10^{-3}

Rapid elimination of the temperature variations will occur if k is small, x is large and the frequency of the oscillations is high. It is seen from the above table that the temperature fluctuations produced by discontinuous control of a few seconds period can be reduced to a negligible amount by using a layer of cork insulation.

Uniformity of temperature at all points inside E will be obtained if the temperature is the same at all points on the surface C. This can be attained by

(1) arranging the heaters A to produce a uniform heating effect at all points of the surface C;

(2) covering the outer surface of D with a material of high diffusivity such as copper, so that any variations of temperature are rapidly distributed; and

(3) covering the inside surface of D with a material of high diffusivity so that any remaining local variations are eliminated.

The above principles were adopted in the design of the thermostats used for this investigation.

§ 2. METHODS EMPLOYED

Two methods of control have been studied, one employing radiant heat and the other a stream of heated air, and it has been found that, provided the above conditions were satisfied, satisfactory results were obtained by either method. Essentially the methods used consist in making C, the control thermometer, one arm of an alternating- or direct-current bridge. The temperature θ of D is kept a few degrees above ambient temperature and the circuit is arranged so that a drop in temperature $\Delta\theta$ (of the order of 0·002° c.) of the thermometer arm at C will switch on the heaters A. These produce a heating effect which will be removed when the temperature of C reaches $\theta+\Delta\theta$. The time of heating of C can be made approximately equal to its time of cooling. If radiant heat is employed, the time lag of the circuit depends on

(1) the time taken for A to reach a temperature sufficiently high to emit radiant heat in quantity;

(2) the time constant of the electrical circuit.

The first factor is reduced to a very small fraction of a second if electric lamps are used, but these must be uniformly distributed about C. The second factor can also be made very small by using the electrical methods described later. The final time-lag can be made a small fraction of a second.

In the warm-air method, A consists of electric heaters (wire-grid type) behind which are placed electric fans which force air past the heaters and distribute it uniformly around the apparatus. The time-lag using this method can be made a fraction of a second by using fine-wire grid heaters and efficient fans.

§ 3. ELECTRICAL CIRCUITS

(1) *Direct-current bridge method.* The circuit for this method is shown in figure 3. The resistance thermometer is a single-layer coil of No. 44 S.W.G. silk-covered copper of 200 ohms resistance wound round and in close contact

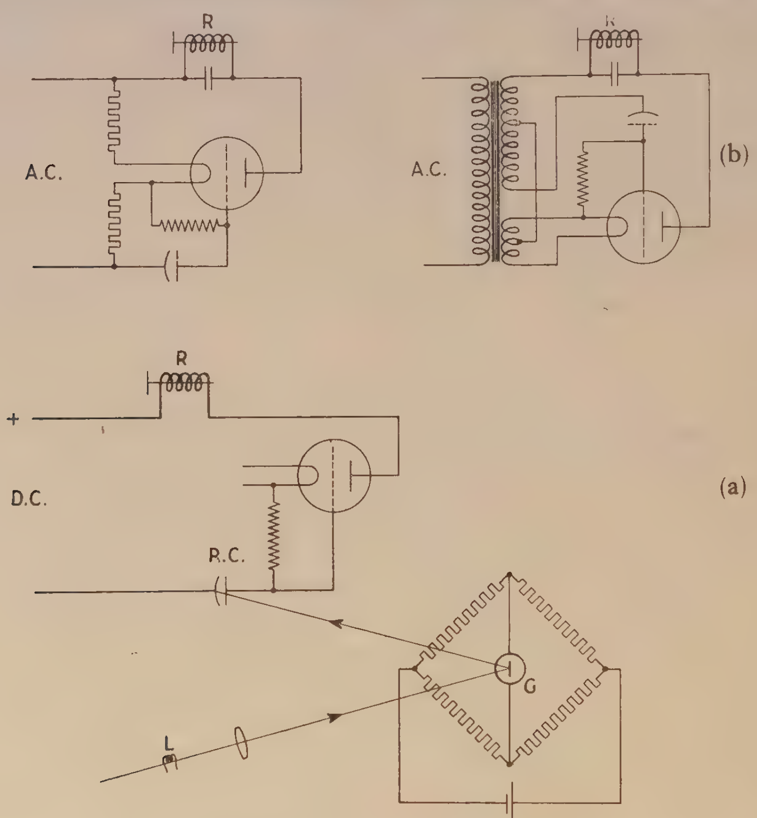

Figure 3. (a) D.C. bridge and photoelectric cell circuits.
(b) arrangements for A.C. supply to valve.
G, galvanometer mirror ; P.C., photoelectric cell ; R, relay ; L, galvanometer lamp.

with part of the apparatus, the temperature of which is to be controlled. This resistance is connected to an equal arm bridge, the other arms being manganin coils. A single accumulator and a reflecting galvanometer are used. The

galvanometer should have a stable zero, but it need not be very sensitive. A type found quite suitable was a Tinsley 576-ohm galvanometer which gave a deflection of 150 mm. per microampere. This gave a deflection of 1 mm. for 1/1000° change in temperature. A 921 R.C.A. photoelectric cell and a 1000-ohm mercury relay were used in the circuit. The voltage supply to the valve can be supplied by direct or alternating voltage. Two simple arrangements for using the latter supply are shown in figure 3. In both cases the maximum anode current is one half that which passes when direct voltages are used, as current only passes when the plate is positive with respect to the filament. The circuits shown in figure 3 were found to be very convenient.

Attached to the relay are two or more lamps which are connected to the electric light supply. The lamps are placed a few feet from the apparatus so that they illuminate it uniformly. Efficient shielding will correct for any local irregularities of temperature at the surface.

The variable resistance of the bridge is arranged so that, when the bridge is balanced, the temperature of the thermometer will be three or four degrees above that of the room. The temperature of the external surface rises and falls by a small amount $\pm\Delta\theta$ about the fixed temperature θ. With different instruments this varied from 0·002° c. to 0·005° c. The reduction of $\Delta\theta$ below 0·005° c. is unnecessary, as this small fluctuation is rapidly diminished as it penetrates a small distance into the surface of the apparatus. This method was used when radiant heat was used as the heating source.

(2) *Alternating-current bridge method.* A discontinuous method of control was devised, using an alternating bridge with valve amplification. The circuit used was similar to that given by Sturtevant (1938), the chief modification being that the high-power thyratron which gave continuous control was replaced by a power valve or low-power thyratron and relay to give discontinuous control. This simplifies the operation of the circuit, gives a more suitable type of control, and greater sensitivity.

One arm of the bridge consists of the thermometer (190 ohms at 15° c., 44-gauge copper resistance, non-inductively wound) and a variable manganin resistance of 10 ohms, the other arms being each of 200 ohms manganin wire, non-inductively wound. To obtain temperature control, this variable resistance is reduced so that for bridge balance the thermometer temperature must rise a few degrees above room temperature. Maximum current at first passes through the relay and the lamps or heaters are switched on. As the temperature of the apparatus rises, bridge balance is approached and the current through the relay falls until the heating supply is suddenly switched off. The lamps or heaters then operate every two or three seconds. It was found that a change of temperature of about $\pm$0·003° c. in the thermometer arm of the bridge caused a change of 1 ma. in the output current, and this is sufficient to operate the relay.

Comparison of the bridge methods. For the radiant-heat method, both bridges were found to be equally good, although the A.C. bridge circuit was more

elaborate. A very compact D.C. bridge arrangement can be made using a box galvanometer and inserting the photoelectric cell in place of the scale. With the heated-air method, the fluctuations in temperature $\Delta\theta$ are not exactly equal and, using a sensitive galvanometer of long period, the spot might overshoot the photoelectric cell and so produce additional heating without control. An electrical time-switch or some such device must be added to prevent this lack of control causing any damage. With the A.C. bridge method this does not occur, as an increase in temperature of the thermometer arm reduces the current through the relay.

§ 4. CONCLUSIONS

Where efficient thermal shielding can be employed, the radiant-heat method, owing to the rapidity with which heat is transferred from the heater to the apparatus, is sounder theoretically and practically. The main difficulty in this method is in the production of uniformity of temperature in the apparatus, and the methods for obtaining this have been discussed earlier. If control is required over a large volume and little or no thermal shielding is available, the heated-air method is superior, as it produces a more uniform heating effect. For precision results, using this method, it is advisable to protect the apparatus from all sources of radiant heat; windows and lamps, etc., should thus be covered. The thermometer may be placed close to or in contact with the apparatus whose temperature is to be controlled, and the observer should not stand near the apparatus.

The radiant-heat method was used in the experimental determination of the electronic charge, and the temperature inside the apparatus was measured by means of a sensitive fine-wire thermocouple. No temperature variations were detected throughout the tests, the temperature being read to 0·01° c. Using the heated-air method, partially or entirely uninsulated large vessels were temperature-controlled to better than 0·01° c., the temperature for these tests being read by means of a sensitive mercury-in-glass thermometer.

§ 5. ACKNOWLEDGEMENTS

This investigation was commenced during the latter part of 1938 at the Natural Philosophy Laboratory, University of Melbourne, and was continued during the following year at the Physics Laboratory, University of Western Australia. The writer wishes to acknowledge the facilities he has been given at both these universities. The investigation was made possible by the help of a Commonwealth Research Grant.

REFERENCES

Laby, T. H. and Hopper, V. D., 1939. *Nature, Lond.*, **143**, 240.
Callender, A., Hartree, D. R. and Porter, A., 1935-6. *Philos. Trans.* **235**, 415.
Carslaw, H. S., 1922. *Mathematical Theory of the Conduction of Heat in Solids* (Macmillan & Co., Ltd.), p. 47, second edition.
Sturtevant, J., 1938. *Rev. Sci. Instrum.* **9**, 276.
Turner, L. B., 1937. *J. Instn. Electr. Engrs*, **81**, 404.

AN APPROXIMATE SOLUTION FOR THE DISTRIBUTION OF TEMPERATURE OR POTENTIAL WITH CYLINDRICAL ISOTHERMAL OR EQUIPOTENTIAL SURFACES

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British Electrical and Allied Industries Research Association

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ABSTRACT. Dipoles or source-sink combinations are used to give an approximate solution of Laplace's equation when the boundary surfaces comprise equipotential or isothermal cylinders. The method is shown to be convenient for a symmetrical arrangement but liable to become laborious in other circumstances.

THE method of solution for the field between two equipotential or isothermal spheres is well known and has been fully analysed by Russell (1911). If the centre of sphere 1 is O_1 and that of sphere 2 is O_2, then charges or sources are placed at O_1 and O_2, as required, to bring spheres 1 and 2 considered separately to their requisite potentials or temperatures. But the charge or source at O_1 causes sphere 2 to have a variable potential or temperature; accordingly, O_1 is compensated by its image (at the inverse point) in sphere 2 and O_2 by its image in sphere 1. This leaves the images uncompensated, which may be rectified by the 2nd-order images and so forth. Since further, the magnitude of the image decreases with its order, a convergent series is obtained and the problem has an exact solution. In the same manner, but with much greater labour and with additional restrictions, a solution could be obtained for more than two spheres. But in two dimensions, with spheres replaced by cylinders, the images are equal and alternatively positive and negative, so that the series is no longer convergent but oscillates, and is also divergent if there are more than two cylinders or a boundary condition other than at infinity, because the number of images of a given order then increases with the order.

If a solution were possible by means of a series of positive and negative point-charges or sources and sinks, then the resultant image-charge or resultant source would be zero. This suggests the use of a dipole or source-sink combination as a method of approximation, a method which has been found in fact to give a reasonable closeness of fit without undue numerical labour. Mathematically, such a method is valid since a solution of Laplace's equation may be built up from a distribution of point charges and dipoles for any space bounded in such a way as to exclude the charges and dipoles.

An example will make the method clearer. Consider three similar cylindrical isothermal or equipotential surfaces in a semi-infinite medium within which heat is generated at a constant rate of $g/2\pi$ watts per cm. length or which are

charged with a uniform electrification of $\epsilon/2\pi$ coulombs per cm. length, where g is the thermal resistivity (in watt-cm.-°C. units), or ϵ the permittivity of the medium. There are many practical examples of this problem, as with three charged conductors near an earthed plane or three underground pipes.

Let the centre of the system be C; let the centres of the individual cylinders be O, O′, O″ respectively. The figure shows one of the cylinders considered. " Unit " sources of heat at O, O′ and O″ give the correct rate of heat generation, but the circles are not isothermal. If this may be corrected by dipoles, which do not alter the rate of heat generation external to them, these dipoles must by symmetry be situated on CO, CO′, CO″; for example, at D, D′ and D″. If the strength of the dipole is M, the temperature it produces at any point P is Mx/r^2,

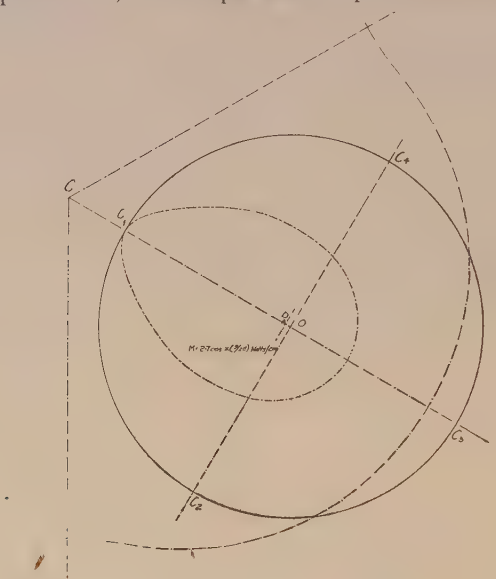

Figure 1.
Solution for OC=5·08 cm., OC=6·8 cm., CD=6·65 cm. and M=2·7 cm. ×($g/2\pi$) watt/cm.

where r is the distance between P and M, while x is the projection of r on the direction of M, and is positive if measured in the same direction as M. In the case mentioned, the directions of the dipoles must be DO or OD, etc. To make the plane surface an isothermal, the images of O, O′ and O″ in the plane surface are occupied by " unit " sinks. To simplify the problem, the distance of the plane is supposed to be sufficiently great for the variation of temperature over the circles due to these images to be neglected, together with the variation of temperature over the plane surface due to the dipoles.

The position and magnitude of the dipoles may then be determined by applying the condition that the temperatures at C_1, C_2, C_3 are equal, which will then also be true of C_4. Whence, if h is the depth of C below ground surface:—

$$\log\frac{2h}{O'C_1}+\log\frac{2h}{O''C_1}+M\left[\frac{DC_1}{(DC_1)^2}+\frac{D'C_1'}{(D'C_1)^2}+\frac{D''C_1''}{(D''C_1)^2}\right]$$

$$= \log \frac{2h}{O'C_2} + \log \frac{2h}{O''C_2} - M\left[\frac{DO'}{(DC_2)^2} - \frac{D'C_2}{(D'C_2)^2} - \frac{D''C_2''}{(D''C_2)^2}\right]$$

$$= \log \frac{2h}{O'C_3} + \log \frac{2h}{O''C_3} - M\left[\frac{DC_3}{(DC_3)^2} - \frac{D'C_3}{(D'C_3)^2} - \frac{D''C_3''}{(D''C_3)^2}\right],$$

where C_1', C_1'', etc., are the projections of C_1 on $D'C$, $D''C$ respectively.

Thus
$$\log \frac{O'C_2\, O''C_2}{(O'C_1)^2} = (K_2 - K_1)M$$

and
$$2 \log \frac{O'C_3'}{O'C_1} = (K_3 - K_1)M,$$

where K_1, etc., $= \dfrac{DC_1}{(D'C_1)^2} + \dfrac{D'C_1'}{(D'C_1)^2} + \dfrac{D''C_1''}{D''C_1^2}$, etc.

The geometry of the problem then permits the determination of M as $af(a/b)$ and CD as $a\,\phi(a/b)$, where a is the radius of the circles, b is OC, and f and ϕ are functions determinable from the equations given. The algebraic expression of these functions is not given here, since it is usually more convenient to measure on a diagram and to solve the equations graphically. The figure shows the solution of these equations for $a=5{\cdot}08$ cm. (2″), $b=6{\cdot}8$ cm., $h=88{\cdot}1$ cm., giving $M=0{\cdot}53a$, $CD=1{\cdot}31a$, i.e. $f(0{\cdot}747)=0{\cdot}53$, and $\phi(0{\cdot}747)=1{\cdot}31$. The broken lines show two isothermals for the case when the dipoles M are omitted; one passes through C_1, while the second corresponds to the temperature attained by C_1 when the dipoles are included. The isothermal corresponding to the temperature of C_1 (and C_2 and C_3) when the dipoles M are included coincides so closely with the circle that it cannot be shown separately on the scale employed. The actual value of the temperature calculated for various points on the circle does not depart by more than 0·4 per cent from the value at C_1, C_2, C_3.

A solution can similarly be obtained for any symmetrical disposition of circles, since the dipoles are located on the lines joining the centre of the complete figure with the centres of the circles. With a completely unsymmetrical arrangement, the method is in many instances too complicated to be worth while, since within each circle is a dipole corresponding to each other circle and situate on the line of centres. For a row of circles, this however reduces to two dipoles in each circle situated symmetrically with respect to the centre on the line of centres, except for the two outer circles, where the outer dipoles are missing.

When the circles are small compared with their axial spacings or distance from the plane surface, the greater accuracy obtained by the dipole method is unnecessary and the principle of superposition may be followed as in a specific example, computed by the present writer (1930).

REFERENCES

RUSSELL, A., 1911. *Proc. Phys. Soc.* **23**, 352 ; 1912. *Ibid.* **24**, 22 ; 1914. *Alternating Currents*, vol. **1**, 2nd edition. (Cambridge.)

WHITEHEAD, S., 1930. *World Power*, **13**, 350.

A ROTATING DIFFERENTIAL PHOTO-ELECTRIC PHOTOMETER FOR PRECISION WORK, by J. T. MacGregor-Morris and A. G. Stainsby *

DISCUSSION

Dr. J. R. Tillman. I was surprised that no mention was made either of methods using A.C. amplifiers or of attempts to do the commutation with thermionic valves. If generous use is made of negative feedback, amplifiers having an extremely high stability against variations of power supplies and giving a very faithful reproduction of wave-form can be made very cheaply. Thus, if two stationary photocells were used, half a cycle out of phase with each other, and receiving light alternately from the variable source and the constant source via, say, a rotating double-sided mirror, each being followed by a good amplifier, the resultant signals could be placed in opposition and the difference, if any, further amplified. A robust meter could now be used. Small differences in the sensitivities of the two cells could be compensated by using a gain control in one amplifier. It might be possible to use only one photocell if the wave-form of the signal obtained could be accurately analysed. The amount of some of the higher harmonics present will change, as the alternate signals (from the variable source and the constant source) vary in relative intensity, by much larger percentages than does the intensity of the fundamental frequency.

I have recently tested a batch of six commercial " rectifier " cells of high sensitivity (480 μa./lumen) and have found good consistency. Thus four of the cells gave sensitivities within 1 per cent of the mean; other properties might of course differ by larger amounts. None the less, I think stress should be laid on one of the inferences to be drawn from Prof. Morris's paper, namely, that it is very difficult to measure, rapidly, small changes in the intensity of a light source, even when use is made of good " rectifier " cells.

Mr. J. S. Preston. Professor Morris has described a method of photometry with rectifier photocells which not only eliminates electrical troubles, but also disposes artistically of all those features of a cell which the photometrist regards as perverse because he can neither predict the magnitude of their effects nor satisfactorily explain their causes. It is, I think, correct to describe this as a new achievement, at least technically, in view of the great difficulties encountered in past efforts to use a similar principle of alternation with the emission photocell followed by amplifiers. Here the power available was larger, but the degree of success attained considerably less, I believe.

The method remains a balancing method, however. It would be of value to discover the true causes of fatigue, so that cell manufacturers might try to

* See *Proc. Phys. Soc.* **53**, 584.

eliminate them, and so enable the rectifier cell to be used with greater precision as a direct measuring instrument, since in other respects it is so admirably suited to this method of use.

Professor Morris mentioned the effect of varying the angle of incidence of the light falling upon a cell. I do not know whether the "obliquity characteristic" has any important effect on the wave-form of the output from his photometer, since the cells rotate. It may be interesting to note that measurements made at the N.P.L. show that all cells coated with a smooth lacquer behave very similarly as regards loss of efficiency with increase in the angle of incidence of the light. This is so irrespective of the appearance, matt or otherwise, of the selenium.

If the amount or flux of light falling on the cell surface is maintained constant as the angle of incidence θ is varied, then the current output of the cell varies as F_θ, Fresnel's value for the fraction of the incident light which is transmitted by a single refracting surface, the appropriate value of the refractive index being about 1·55. Or, of course, if the cell is at a constant distance from a constant light source, and is rotated, the output current varies as $\cos\theta \, . \, F_\theta$. (In both cases the illumination-current characteristic of the cell is assumed here to be linear.) This suggests that the cell is equally efficient for all the light which actually enters the first lacquer surface, irrespective of the angle of incidence. The second interface, namely, lacquer-sputtered metal film, appears to be without influence in this respect, but it may be noted that the light incident on this interface is confined within a cone of semi-vertical angle equal to the critical angle for a lacquer-air surface.

Mr. L. T. Minchin. It may be of interest to mention another name frequently used for these cells*—"blocking-layer". I presume both this and "barrier-layer" are translations of the German name "Sperrschicht".

Do I understand that pairs of commercial cells are not suitable for this method? Is it necessary to have them specially made at the N.P.L.?

Mr. D. W. Stops. I should like to ask Professor Morris how it was ensured that the two photocells employed in his photometric instrument gave equal e.m.f.s for equal sources of light and for equal periods of time.

Lt.-Col. K. Edgcumbe. This paper is of particular interest to photometrists as outlining a method which would seem to combine the well-known advantages of objective photometry with an accuracy even greater than that of the visual. The most serious limitations, although not stated by the authors, would appear to be those due to the colour response of the cells themselves. It is likely, however, that when colour filters are used, a precision comparable with that of the visual method might still be possible.

* In presenting the paper, Prof. Morris mentioned that the names *Vorderwand* and *barrier-layer* are both in use for this type of cell. [Ed.]

The authors stress the point that their chief objective was to obtain relative readings with a rapidly varying source, viz., an electric arc. If this was all, it would appear that a photo-electric cell connected to a galvanometer would have given the information required and would have enabled a curve such as that shown in figure 9 to be drawn with considerably less complication. It appears to me that the chief field of usefulness for this method will be the comparison of two similar light sources to a high degree of precision.

Turning to the method itself, my own feeling would be that the advantages of using a single cell would far outweigh the difficulties, and as regards the trouble experienced, due to the heavy alternating current, it would seem that it should be possible to damp this out by the use of a high-capacity condenser. In any case, I think the authors should use the current output through a low resistance as the criterion, rather than the voltage. They appear to assume that the only advantage of the current method lies in its increased sensitivity, but they would find that many of the troubles, particularly those due to temperature and drift, are considerably reduced and that almost perfect linearity is possible with a moderate galvanometer resistance. In this connection, the bending over of the curve of figure 6 seems to show that the cells are being used under adverse conditions.

The differentially wound galvanometer discarded by the authors would seem to be the simplest way of applying the current criterion; but if this is undesirable, then the joining of the two cells in series, with the galvanometer so connected that it carries the difference current only, should not be difficult. As commutation would always occur during the period of zero illumination, the fact that the circuit was periodically opened should have no effect upon the performance of the cells.

This latter consideration leads me to ask whether the authors have found that the characteristics of the cells are, in general, the same when subjected to a rapidly varying illumination, as they are with a steady illumination of the same average value.

AUTHORS' reply (by Prof. MACGREGOR-MORRIS). Mr. Tillman has described a neat way of doing away with the use of a commutator, and in the hands of one experienced in the precision use of such apparatus, it is practically certain to give as good results as ours. As to the question of relative costs, I am doubtful ; and I naturally use devices with which I know the traps and how to overcome them. Other experimentalists more conversant with the use of amplifiers and similar apparatus—though they have their defects—might well get as good results, and that without any noise or vibration.

Mr. Preston's encouraging remarks will certainly help us to persevere. My only doubt is whether he has found such similarity in behaviour that the loss of efficiency with the increase in the angle of incidence of the light is unaffected.

In reply to Mr. Minchin, as these were the first cells to be used for this

purpose, and since I had friends there, I decided to go the N.P.L. in the first place, and the cells have turned out exceptionally. Now, however, that more experience has been gained by manufacturers, I would certainly turn to them.

In reply to Mr. Stops, the method is given in the paper, and it has to be remembered that our apparatus turned at 10 revs. per second, and so the length of time is very short.

Lt.-Col. Edgcumbe's remarks are very much to the point. I fully agree that there are other fields where this instrument will have much wider application. But the simple arrangement outlined by Col. Edgcumbe was used by us until the repeated determination of all the corrections became so tedious that for the sake of time we were forced to develop the method described in the paper, and it has certainly saved much time and has added to the accuracy of our results. Turning to his next point, the principle involved in using a large-capacity condenser (if it is perfect) is sound, but my experience of the lack of perfection, unless a very good one is used, made me decide to put this method on one side. It is possible that I am out of date in this matter.

The cells were actually used at 10 f.c. usually, and it is the bottom graph in figure 6 which applies (i.e. the Barnard cells), where the graph is not far from straight.

REVIEWS OF BOOKS

The Second Yearbook of Research and Statistical Methodology Books and Reviews, edited by OSCAR KRISEN BUROS. Pp. xx + 383. (The Gryphon Press, Highland Park, New Jersey, 1941.) $5 (less 10 % if ordered direct from the publishers.)

Dr. Buros is disturbed, and rightly so, because text-books dealing with statistical and allied subjects often betray the ignorance of their authors in the recent developments of that subject. He argues that if the writers are ignorant, those teachers who do not publish are likely to be in even worse case ; and if the teacher is behind the times, what about the pupil ? To ameliorate this position, Dr. Buros had the idea of collecting together the titles of books on the subjects concerned, and printing a collection of reviews of each book. Among the purposes so served are (1) that those conscious of their defects could find the names of books for study, and could receive guidance as to which were good and which bad ; (2) presumably the bad books would be pilloried ; (3) the fact that some reviewers are woefully ignorant of the subject, or else too kind or too polite to give frank reviews, would be exposed. That this happens is, I think, undoubted. That it is worth a whole book to illustrate it is very doubtful.

So much for the project. Next, for its accomplishment. In 1938, Dr. Buros brought out a work, covering, if I have understood him correctly, the books reviewed between 1933 and 1938. The second yearbook, the one now under review, deals with 359 books published from 1933 onwards and contains 1652 reviews or parts of reviews of them. The prospective purchaser will naturally ask, " What books, and what reviews ? " The author's answer is that in the first edition he had attempted to include " methodology books in all fields ", and he now says " Some classes of methodology books (e.g., books on microscopy and the philosophy of science) have been excluded." Perhaps the first few titles will give as good an idea as any other explanation. They are :—Statistical Mathematics, The Nomogram, Mathematical Analysis for Economists, Family Expenditure, Educational Research, The Technique of Marketing Research, Time-series Charts, Bibliographical Citation in the Social Sciences, Graphs, Statistical Methods, Practical Application of the Punched card method . . ., The Preparation of Reports, A History of Historical Writing, and The Study of Society.

As to the choice of reviews, the author has no criterion of selection by merit. He has a list of 283 journals, and includes excerpts from all reviews of a book which appear in any of these journals. Thus we find Pledge's *Science since 1500* reviewed in *Isis*, the *Journal of the Royal Microscopical Society*, *Nature*, the *New Statesman*, *Philosophical Magazine*, our own *Proceedings*, *Science and Society*, the *Spectator*, *Time and Tide* and the *Times Literary Supplement*. Surely a library could risk 7*s*. 6*d*. without this weight of testimony ? The only reviews of Burington's *Handbook of Mathematical Tables* (the name is wrongly given as Burlington in the heading and in the catch-word at the top of the page, but correctly in the index) are from the *American Mathematical Monthly*, the *Journal of the American Society of Agronomy*, *News Edition of the American Chemical Society*, and *Popular Astronomy*.

Since only excerpts are given from the reviews, the question of their accuracy of

representation arises. I find four of my own reviews, and I am only slightly misrepresented in one place. Dealing with Miller's Tables for converting rectangular to polar co-ordinates, I expressed the view that the scheme for ascertaining the quadrant in which the radius vector lies had no advantage over the direct deduction from first principles ; the omission of a few words makes it appear that I thought the *whole* scheme had no advantages over an *ad hoc* calculation in each particular case.

In the introduction, the editor mentions possible improvements in future editions. Of these, the most important seems to be the inclusion of books published in languages other than English, for many of the most important works are necessarily omitted under the present restriction. The inclusion of foreign books will, however, have the effect of swelling the volume considerably, and attention might well be given to the question of defining more clearly the field covered and of pruning vigorously the list of journals from which reviews are taken.

There is also a suggestion that articles, as distinct from books, might also be reviewed in future editions. This plan has much to commend it, though it might be still better to let experts give a more general review of progress, taking into account those papers which they regard as important. No doubt experience will show the best form for these articles to take.

There is an elaborate system of indexing—lists of journals from which reviews have been taken, an index of reviewers and indexes of the books reviewed. I have enjoyed reading the book, or bits of it selected at random, when perhaps I ought to have been doing something more active, but I am not certain that it would be everyone's taste.

J. H. A.

Practical Solution of Torsional Vibration Problems, Vol. 2, by W. KER WILSON. Second Edition. Pp. xx + 694. (London: Chapman and Hall, Ltd., 1941.) 42*s*.

The first six chapters of this very useful work were included in vol. 1, whilst vol. 2 comprises six further chapters, as follows :—Chap. 7, Determination of Stresses due to Torsional Vibration at Resonant Speeds ; Chap. 8, Measurement of Torsional Vibration Amplitudes and Stresses ; Chap. 9, Analysis of Torsiograph Records ; Chap. 10, Torsional Vibration Damping Devices ; Chap. 11, Rotating Pendulum Vibration Absorbers ; Chap. 12, Dynamic Characteristics of Electrical Generating Sets Direct-coupled to Internal Combustion Engines, and Flywheel Calculations for Marine Installations.

Broadly, vol. 2 treats of the subjects of damping, the measurement of vibration and means for the elucidation of these measurements. The most recent developments are included, as, for instance, the strain-gauge technique, which promises to pave the way for rapid progress in the practical measurement of vibration stresses in airscrew blades. There is also a fairly extensive treatment concerning the utilization of rotating-pendulum devices to overcome specific orders of resonant torsional vibration in the running range.

The strain-gauge procedure may be said to be in its infancy, and there is some question as to the accuracy of the results in its present stage. The author gives a description of an electrical method which is grounded on strain-sensitive resistance strips, which strips respond to changes in the surface strain at selected points on the blades. The rotating pendulum " damper " has the remarkable property that it can be made to nullify the effect of a particular order of forced torsional vibration. Care must, however, be taken to ensure that other orders previously innocuous do not become

accentuated. As with practically all anti-vibration devices, it alters the original system by modifying its frequencies and modes of vibration and, in addition, each pendulum employed introduces an extra frequency.

As a compendium of the various notions and devices pertaining to torsional vibration which have become crystallized, the author's work may be classed as comprehensive. There is even included in volume 2 a *Selected List of Patents relating to Vibration Study*. In the placing of these devices in their true perspective it must be realized that an artifice which will diminish vibration in one set of circumstances may increase it in other cases, or at other speeds in the same operating range. So far as is possible, vibration characteristics should be taken into account in the design stage. If the trouble and its main cause are clearly understood it may be found that appropriate alterations in the dimensions of a particular shaft, for example, may be all that is called for, whereas the insertion of a so-called vibration damper may have little influence in eradicating the trouble, and may even amplify it.

Certain items dealt with in the book are from their very nature empirical and imperfectly understood, as, for instance, elastic hysteresis and apparent damping in engine crankshafts. Care must be taken to ensure that a particular effect is not over-estimated whilst other equally or more important effects are neglected. For example, it has become customary to calculate the dissipation due to hysteresis, which may be only 10 per cent of the total dissipation, and to multiply it by an empirical factor determined so as to make the calculated torsional amplitude coincide statistically with the measured one in a number of installations (Den Hartog).

Torsional vibration is only one part of the practical story. Resonant lateral vibration can, in certain cases, be more serious than torsional, and frequently the two are indissolubly coupled.

Like the first, the volume under review is copiously illustrated, both diagrammatically and photographically. J. M.

Visibility in Meteorology, by W. E. KNOWLES MIDDLETON. Second Edition. Pp. x+165. (Toronto, Canada: The University of Toronto Press; London: Sir Humphrey Milford at the Oxford University Press, 1941.) $2.50.

The appearance after six years of a second edition of this monograph is a sign that Mr. Middleton has supplied a want and that he has done so in a not unsatisfactory fashion. The author has, in fact, produced a stimulating account of researches and meteorological practices bearing, as he himself puts it, on the question, "How far can I see to-day?" Despite certain shortcomings of the work, all interested in the visual range of objects and lights by day or night will find Mr. Middleton's book of value.

Compared with the first, the second edition contains additional matter on the optical and other properties of aerosols and on the perception of lights and brightness contrasts under different conditions. The chapter on instruments is amplified and a much extended discussion is given of practical methods of observing visual range and of recording and analysing the results. The forecasting of fog forms the subject of a new chapter. An adequate theory (due to the author) of the visual range of coloured objects is included as an appendix and the bibliography is increased from 146 to 342 references.

The theory of the air-light bulks largely, and the author rightly explains in some detail Koschmieder's rather complex analysis. It is a pity, however, that the simplest proof of Koschmieder's main result has been overlooked. The apparent brightness H_s of a small black object consists simply of the air-light of the layer of atmosphere

between observer and object. If the object is removed the observer sees, added to the air-light, the brightness H_h of the sky beyond the object, reduced by the transmission factor $e^{-\sigma l}$ of the intervening atmosphere. Thus, provided the sky brightness does not change as the observer moves up to the position of the object, keeping his direction of view the same, we have $H_h = H_s + H_h e^{-\sigma l}$, which is Koschmieder's result.

If the object is not small, then on its removal the air-light will be increased because rays which previously were obstructed by the object will reach and be scattered in the intervening atmosphere. On this account, Koschmeider's expression gives too large a value for the apparent brightness of the black object. Mr. Middleton refers briefly to Foitzik's work on this question but gives more attention to the complication which ensues when the object is so small that diffraction of light round the edges may play a rôle. Löhle's theory of this edge effect is certainly not easy to follow, but Mr. Middleton reproduces it uncritically and derives a formula for the apparent brightness at the centre of a black object, which involves explicitly neither the distance of the object nor the coefficient of extinction. The uninitiated reader is left puzzling how Löhle's formula is linked with Koschmieder's result. This is a good example of the main defect of the work, the author's tendency to present condensed accounts of theoretical or experimental researches without adequate welding of these members into a single structure. Some needless repetition, as well as lack of continuity, are the inevitable consequences of this tendency.

Of the few minor errors and obscurities, two will be mentioned. The table on p. 90 includes values for the visual range of objects seen against the earth's surface, computed from formula 5.16. This formula, however, applies only when object *and* background present vertical surfaces. When the background is horizontal, the formula must be modified by the substitution of $2R_2$ for R_2. On p. 48, Holladay's glare formula is quoted with the numerical value 10 for the constant K. It should have been stated that the units implied are candle/sq. ft. for brightness, foot-candle for illumination and degree for angle; the first two of these are not the units commonly employed in the rest of the book.

The author is at his best when giving his own views, as in the appendix, and when he is not following his authorities too closely. He might bear this point in mind when preparing the further edition which in due time will surely be required. W. S. S.

Practical Physical Chemistry, by ALEXANDER FINDLAY. 7th Edition. Pp. x + 335. (London: Longmans, Green and Co., Ltd., 1941.) 12*s*. 6*d*.

In such a large and ever expanding field as that of practical physical chemistry, no single book can satisfy the likes and dislikes of every individual reader. It is, however, probably true to say that Professor Findlay's book meets the varied demands of the subject more successfully than any other English text.

The present edition follows closely that adopted in earlier ones, and it is a tribute to the original planning that the new matter has fallen into place without destruction of the logical sequence. The book has, since the last edition in 1935, been revised and enlarged ; the use of small type in certain places has enabled this to be carried out without greatly increasing the bulk. The production of the book has vastly improved, and it has become correspondingly more readable. The use of bold type for the general constants is to be commended in that it familiarizes the student at the start with the present system of scientific nomenclature.

New material in the present edition includes : the use of the thyratron relay, the

vapour-density methods of Weiser and of MacInnes and Kreiling, the maximum bubble-pressure method of measuring the surface tension of liquids, the use of the antimony electrode in determinations of hydrogen-ion concentration and of the tungsten electrode in redox systems, boiling-point curves of binary liquid mixtures and equilibrium curves in three-component systems. The number of experiments and of applications of experimental methods described has been increased. References to the literature have largely been brought up to date, as has some of the text itself—for example, the discussion of surface tension and of viscosity in relation to constitution.

A few of the chapters still remain rather unsatisfactory. The treatment of colloids, of surface phenomena and of reaction velocity are lacking both in appeal and in completeness. Also there is no direct experimental study of the kinetic theory. It is to be hoped that a later edition of this deservedly successful book will remedy these deficiencies. R. F. B.

Diffusion in and through Solids, by RICHARD M. BARRER. Pp. xiii+464. (Cambridge: The University Press, 1941.) 30*s*.

A great deal of the progress in science is due to the painstaking work of scientists who undertake the task of reviewing, systematizing and tabulating the results of experiments. A book on a subject not hitherto dealt with in such detail is sure of a welcome.

As the measurement of permeability and diffusion constants forms the main substance of this book, the author gives details of the solution of the diffusion equation needed in the discussion of typical experiments. The flow of gases through narrow channels is also discussed, with its application to flow through porous materials. A series of chapters are devoted to the absorption of gases in, and diffusion through, metals and alloys, including the very interesting hydrogen-palladium and deuterium-palladium systems. Less well-known aspects also dealt with include the permeability to gases and vapours of complex silicates, and of complex organic substances—rubber, resins, leathers, paper, etc. Inter-diffusion and self-diffusion in the case of metals, and ionic diffusion in the case of alkali and silver halides, form the subject matter of special sections. A brief account is also given of experimental and theoretical work on the mobility of atoms on surfaces.

For a proper appreciation of the processes of diffusion it is necessary to marshal a large array of facts from many branches of physics. The structure of complex organic and inorganic substances, the nature of the imperfections in otherwise "perfect" crystals and facts about F-centres and U-centres (in ionic crystals) form examples of such subsidiary matter.

The theoretical side is, in general, adequately summarized and the documentation is very complete. A special feature is the collection into tables of a great mass of experimental data, which should prove of great value to physicists and chemists alike. M.B.

The Birth and Death of the Sun, by GEORGE GAMOW. Pp. xiv+238. (London: Macmillan and Co., Ltd., 1941.) 12*s*. 6*d*. net.

This excellent book gives a very clear and interesting account of the latest views of the relevance of nuclear physics to the problems of stellar constitution and evolution. At last a plausible idea of the mechanism by which stellar energy is made available for radiation can be formed, and Professor Gamow here explains it in a form suitable for the ordinary non-scientific reader. A noteworthy feature of the presentation is the

provision of many original and ingenious diagrams, which give a very vivid idea of the rather abstract processes described. From the layman's point of view the book is slightly marred by a tendency to present useful hypotheses as established facts, and a somewhat excessive optimism (or should we call it pessimism ?) which leads the author to declare, for example, that " it may be hoped that in the course of the next few years a satisfactory solution of this last remaining puzzle of stellar evolution will finally be found ". Physicists of experience will not attach too much weight to this, and the book can be recommended to them almost unreservedly as the best description we know of the present state of the matters discussed. This having been said, it will not be out of place to point out a few defects, with the remark that their importance is negligible compared with that of the merits of the book.

It is not true that " it was first indicated by the Russian chemist, Dmitri Mendelyeev " (p. 48) that the elements show a periodicity in properties. Newlands indicated this before Mendelyeev, but his work met with a cold reception. Plate VII (p. 126) is the worst of many bad illustrations which exist of the Harvard sequence of stellar spectra. Not only is it not stated that most of the spectra shown are of the iron arc and not of stars, but the stellar spectra themselves show next to nothing of the characteristic features of their types. Capella is not a typical red giant (p. 141), nor do " the bright emission lines of novae show a conspicuous shift toward the violet end of the spectrum " (p. 185). They are broadened but undisplaced, and the absorption lines at their edges show the shift. The disc-like form of the stellar system was, in a sense, " proposed by Herschel more than a century ago " (p. 207), but it was clearly described by Thomas Wright, of Durham, nearly two centuries ago. Finally, we know of no evidence that the two nebulae apparently near the great Andromeda nebula are " satellites " of the latter (p. 218). H. D.

Weather Analysis and Forecasting. A Textbook on Synoptic Meteorology, by SVERRE PETTERSSEN. Pp. xvi + 505. (London: McGraw-Hill Publishing Co., Ltd., 1940.) 35*s*.

The concepts *air mass*, *front* and *wave cyclone* to-day form the framework of the weather forecaster's mental processes. They represent, so to say, the leitmotifs of the weather tone poem, give it line and form where the untrained ear finds only notes strung together rather incoherently. As yet, the structure of the piece is by no means thoroughly understood, but at least the stage has been reached when it is worth while for the critic's analysis to be given, and, very fittingly, one from the Norwegian school of meteorologists, who have been mainly responsible for the development of the concepts and their interplay, has now undertaken the first thoroughgoing presentation of the analysis. For thoroughgoing this book attempts to be, and is not to be compared with a number of introductory treatments of synoptic meteorology—the meteorology of the weather map—already provided. It seeks to present a unified picture of the laws of dynamics and thermodynamics working themselves out on the macroscopic scale in the weather sequence, day by day. Full mathematical treatment is accorded where meteorological thought is sufficiently clarified to allow this ; some readers indeed will feel that the mathematical horse has been ridden too hard, not because of any difficulty in the mathematics, but because its results often seem of doubtful validity in a subject in which to know what to neglect and what to consider is half the battle.

The book opens refreshingly by a chapter headed *Air Mass Characteristics*, followed by *Stability and Instability in Relation to Weather Phenomena*. These 137 pages show

the author at his best, though many meteorologists will not assent to all that is in them. They are pointed by excellently-conceived diagrams—a feature indeed of the whole book—giving succinct presentation of the physics of a multitude of weather phenomena, and more than mere illustration. A more pedestrian but necessary chapter on the production and transformation of air masses follows, and then we pass to the kinematic aspects of weather: wind and the formation of fronts, with a discussion of the latter's characteristics. The kernel of the book is then reached with 50 pages devoted to waves and cyclones, and here least is understood, so that a completely successful treatment is not at present possible. Some integrating physical idea is still awaited, and that in turn may wait on more data being acquired in a more purposeful way.

The remaining four chapters deal with the analysis of weather charts and the methods of weather forecasting. Surface and upper-air charts and the vertical cross-section are each considered, and Namias, an American meteorologist, contributes one of these chapters, entitled *Isentropic Analysis*, showing how a suggestion made by Sir Napier Shaw many years ago is beginning to show some fruit. Petterssen's personally devised kinematic method is not perhaps worth all the space devoted to it—meteorological data and the forecaster's time are rarely sufficient to enable second and third order differentials of pressure with respect to time and position to be evaluated. But the last chapter, giving examples of charts analysed, has its feet on the ground, and the value of many of the 31 concisely expressed rules, previously derived, is well demonstrated.

In this book much has been attempted and much done. No meteorologist can afford to neglect it, and it should find a place on the shelves of all university libraries. It may well do much to increase the interest of physicists generally in the subject and, if so, will prove of inestimable value, for meteorology with its many problems needs greatly to recruit the interest of first-rate minds. But because it may largely spur the minds of readers ready to be enlightened and stimulated, it is the more regrettable that the book gives an impression of hasty compilation, with too dogmatic statement in many places, and on p. 133 it is implied that radiation exchange processes in the atmosphere may on occasion violate the second law of thermodynamics. A second edition will, however, surely be called for, when it is hoped the author will remove such blemishes as now appear. At the same time the publishers, who have made a generally handsome volume, should find a more adequate method of representing surface weather maps ; those given leave practically all the weather data illegible.

P. A. S.

Edmond Halley as Physical Geographer, and the Story of his Charts, by S. CHAPMAN. Pp. 15, with 6 figures. (London: Royal Astronomical Society, 1941.) 2*s*. 6*d*. net.

This little monograph recalls to us that Halley is entitled to fame not only as an astronomer but also as the originator of that graphical charting of geophysical data which is now a commonplace in meteorological and geomagnetic studies. Prof. Chapman's title is modest in its claims : not only does he trace the history of Halley as explorer and cartographer, but he also gives us excellent reproductions of three of Halley's meteorological and magnetic charts. A modern map has a perpetual interest for anybody with imagination ; how much more fascinating are these charts, dating back to the times when

" Geographers on Afric maps
With savage pictures filled their gaps ",

when the S. Atlantic could be described as the Aethiopick Ocean and the Amazon boldly linked with the Plate! Not that the interest of the charts is entirely historical: Prof. Chapman shows how modern rectification of Halley's longitudes and incorporation of contemporary data not known to Halley have given us " reasonably reliable knowledge of the distribution of the magnetic declination over a large part of the earth in A.D. 1700 ".

A. H.

Electro-Magnetic Theory, by J. A. STRATTON. Pp. xiv + 615. (London: McGraw-Hill Publishing Co., Ltd., 1941.) 42*s*.

In recent years the electromagnetic theory has been applied in fields of research hitherto comparatively little known or, at any rate, studied only for their academic interest. The results of research have led to wide applications of wireless waves in directions beyond their use in broadcasting, to subjects of engineering importance such as antenna design, transmission lines and to cavity oscillations. These new problems call for a change in the study of the principles of electromagnetism and for stress upon branches of it which have hitherto received but scant treatment. They have, in fact, been omitted from textbooks until the last year or two.

These requirements have been met to some extent by a series of publications in the International Series in Physics, and an outstanding contribution is presented in this volume by Professor Stratton.

This work assumes a sound knowledge of the general principles of electricity and magnetism, including the general properties of circuits and some knowledge of the practical aspects of thermionics. It may be said to anticipate that the reader has followed a course of study such as terminates in this country with the introduction to Maxwell's equations. The book begins in the first few lines with the postulation of these equations, so that the reader should know something of Faraday's laws of electromagnetism and their quantitative expression.

The avowed object of the presentation of the subject is to lay less stress upon the study of the stationary state which characterizes Maxwell's treatise and many later works which followed Maxwell's example; a more complete account of variable fields is thus included. The m.k.s. system of units is adopted; i.e. the meter-kilogram-second system with the coulomb as the unit of charge, and the system is explained together with its relation to other systems of units. These units appear to have been adopted more widely in America than in this country, where most workers are accustomed to the mixed system. It would, however, appear that one of the minor changes to which we must accustom ourselves is the spread of this system, which the author points out was first suggested by Maxwell and is not the work of subversive engineers.

In spite of an extended treatment of modern applications, the fundamentals of the theory are fully and carefully presented. Short cuts are avoided and the more subtle and difficult points are adequately dealt with. A little less than half the book is devoted to fundamental aspects of the theory and includes chapters on the field equations, upon field energy and stresses, and upon the electrostatic and magnetostatic fields.

Three extensive chapters on electromagnetic waves follow, and these are devoted to plane, cylindrical, and spherical waves. They include an account of Bessel and spherical harmonic functions such as is required for the purpose of the book. These are treated with a completeness which distinguishes the presentation from a number of works on the subject, and in this respect the author has provided a very useful work of reference.

The book is essentially a theoretical work, but the last two chapters are devoted to practical problems of the highest importance in modern application of electromagnetic theory. These include the well-known derivation of the laws of reflection and refraction at dielectric boundaries and of Fresnel's intensity relations, together with the theory of total reflection and of metallic reflection. Subjects which have been hitherto to some extent or almost entirely omitted from textbooks, such as the theory of the skin effect, waves in hollow pipes, oscillations in a spherical cavity, and the effect of the earth on the propagation of radio waves receive, if not full, at least stimulating attention.

The work is one that will be found continually useful to those who require a sound knowledge of the large-scale phenomena of electricity and magnetism, and the book will be found useful both as an advanced textbook and as a work of reference. H. T. F.

Tables of Physical and Chemical Constants, by G. W. C. Kaye and T. H. Laby. Ninth Edition. Pp. 181. (London: Longmans, Green and Co., 1941.) 18*s*. net.

Apart from an expression of regret at the untimely death of the senior collaborator, it is difficult to find anything new to say about a work so long and firmly established as "Kaye and Laby". So far as present conditions permit, the ninth edition has been brought up to date, but the general character of the book is still that of its 1911 prototype. It is a handy and unpretentious compilation of all the less esoteric physical and chemical constants likely to be required by the average non-specialising physicist.

It is all too easy, in reviewing a work of this kind, to point out deficiencies in selected sections—for instance, X-ray spectral data are quoted only in wave-lengths, and not in wave-numbers, and no indication is given of the transitions to which the emission lines correspond. Such minor deficiencies are, however, mainly if not entirely due to shortage of space, and it would be a much harder task to get a more generally useful set of tables into the same bulk than it is to emit criticisms like this. H. R. R.

A Text-Book of Electricity and Magnetism, by G. R. Noakes, M.A. Pp. x + 513. (London: Macmillan and Co., Ltd., 1941.) 8*s*. 6*d*.

Although the scope of this book has been mainly decided by the needs of candidates for Higher School Certificate and University Scholarship examinations, it includes much that is outside such syllabuses, this extra material being distinguished as a rule by the use of more closely set type than the rest. Careful attention is given to sources of frequent difficulties and errors and to the graphical representation of experimental results. Examination questions are appended to the chapters, and the text is abundantly illustrated with 311 line diagrams and photographs. The calculus is used throughout.

One of the most striking and most welcome features of the book is its modern outlook; up-to-date information is given in the descriptions of the more recent developments, and current ideas are freely used in explanations of fundamental principles. The electron-volt, for example, is introduced as early as p. 8, and the Van de Graaff generator at the end of the first chapter.

Electrostatics is treated in the first two chapters, and magnetism in the next two, and in three later chapters (XIV, XXI and XXII dealing with magnetic materials, ferromagnetism and geomagnetism); current electricity occupies ten chapters (V–XIII and XV), which are followed by one on units and dimensions, one on conduction in gases and five (XVIII–XXII) on more recent developments. Amongst the topics

outlined in these later chapters are: determination of electronic charge, mass spectrographs and isotopes, electron tubes (thermionic valve, cathode-ray oscillograph, photoelectric cell, electron microscope, X-ray tube), quantum theory, atomic structure and spectra, radioactivity, nuclear structure and reactions, high-voltage generators, and cosmic radiation.

The Higher School Examinations are as much a test of the schools as of the individual candidates; and there need be little wonder at the high standards attained by candidates from some of the schools if the teaching is on the plane of this first-rate book by the senior science master at Giggleswick and his companion work on Light, which has already been reviewed in these *Proceedings*. It is, in fact, one of the best, if not the best, of the electrical text-books not only for schools, but also for university students in their first and second years. As an introduction to their more advanced studies it can be unreservedly recommended; it is excellent value for a very modest price. W. J.

Wave Motion and Sound, by F. C. CHAMPION. Pp. 67. (London: Blackie and Son, Ltd., 1941.) 5*s*. net.

This book is Part IV of the publisher's *University Physics* series for first- and second-year courses at a university.

Wave motion and the various aspects of sound, such as its generation, intensity, velocity and frequency, are dealt with in the relatively small space of fifty-odd pages. In consequence, the subject matter suffers in places from inadequacy of treatment, and one feels that, to remedy this defect partially, the space allocated to the examples at the end of each chapter could have been used to better purpose. Line diagrams are employed throughout, and on the whole these are well executed, but the clarity of the text is marred in places by errors in punctuation and composition: on page 23 the end correction for the resonance length of a cylindrical tube is wrongly quoted. In the reference to supersonic generators in Chapter IV it is surprising to find no mention of the magneto-strictive oscillator.

The answers and hints for solution of the examples at the end of the book should prove particularly helpful to students. R. W. B. S.

RECENT REPORTS AND CATALOGUES

Mechanical Behaviour of Bitumen, by W. Lethersich. (Technical A/T 83, 1941.) Pp. 28. British Electrical and Allied Industries Research Association, 15 Savoy Street, London, W.C. 2. 15*s*.

Researches on the Structure of Alloys, by W. Hume-Rothery, M.A., D.Sc., F.R.S. Research Report, R.R.A. 562, June 1941.) Pp. 12. British Non-Ferrous Metals Research Association, Euston Street, London, N.W. 1. 2*s*. 6*d*.

Electrochemical Analysis Apparatus. (Leaflet GT 1325, August 1941.) Griffin & Tatlock, Ltd., Kemble Street, London, W.C. 2.

Books in New Condition at Reduced Prices. (Catalogue 595, November 1941.) Pp. 40. W. Heffer & Sons, Ltd., Cambridge.

Safe Handling of Radioactive Luminous Compound. (National Bureau of Standards Handbook H 27, May 1941.) Pp. 15. U.S. Department of Commerce, Washington.

Testing of Timepieces, by R. E. Gould. (National Bureau of Standards Circular C 432, August 1941, superseding C 392.) Pp. 27. U.S. Department of Commerce, Washington. 15 cents.

Safety Rules for the Installation and Maintenance of Electric Supply and Communication Lines. (National Bureau of Standards Handbook H 32, September 1941, superseding H 10.) Pp. 177. U.S. Department of Commerce, Washington. 65 cents.

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7. *Special Publications*, critical monographs and reports on special subjects prepared by experts or committees, are issued from time to time. Particulars of some of these publications are given in the advertisement pages of each Part of the *Proceedings*.

MEETINGS

At approximately fortnightly intervals throughout each normal session, meetings are held for the reading and discussion of papers, for lectures, and for experimental demonstrations. Special lectures include: the *Guthrie Lecture*, in memory of the founder of the Society, given annually by a physicist of international reputation ; the *Thomas Young Oration*, given biennially on an optical subject ; the *Charles Chree Address*, given biennially on Geomagnetism or a cognate subject ; and the recently founded *Rutherford Memorial Lecture*. A Summer Meeting is generally held each year at a provincial centre, and from time to time meetings are arranged jointly with other Societies for the discussion of subjects of common interest.

SUBSCRIPTIONS

Fellows pay an Entrance Fee of £1. 1*s*. and an Annual Subscription of £2. 2*s*. Student Members pay only an Annual Subscription of 10*s*. 6*d*. No entrance fee is payable by a Student Member on transfer to Fellowship.

Further information may be obtained from the Honorary Secretary (Business) at the office of the Society,

1 Lowther Gardens, Exhibition Road, London, S.W.7